CHAPTER 11

MAINTAINABILITY DEMONSTRATION PLANS

CONTENTS

1 INTRODUCTION

1.1 The purpose of conducting a Maintainability Demonstration is to provide evidence that a specified Maintainability parameter (e.g. MTTR, MART, MDT, etc.) will be attained during operation. This is achieved by undertaking a demonstration test where maintenance tasks are performed and the time required to complete the task is recorded. The data collected is used to determine whether the Maintainability is acceptable. This decision is reached once a significant number of tasks have been performed; this number is governed by the Maintainability parameter being demonstrated, and the particular test method chosen.

1.2 The demonstration test is defined by one or more numerical requirements and risk levels that govern the decision criteria of the demonstration test. There are many test methods available, each with a different specification for the following:

a) Type of maintainability parameter.

b) Accept and reject criteria.

c) Associated risk levels.

Test methods are detailed in MIL-HDBK-470A[1], BS6548 : Part 6[2] and the maintainability demonstration model MDEM[3] which supports NES 1017[4].

1.3 The basis for test methods is hypothesis testing, which is described in PtDCh7. Typically, each test method has a null (H_0) and an alternative (H_1) hypothesis, and producer's (α) and consumer's (β) risks. For example, a test specification might be:

H_0 : Mean Active Repair Time = 30 minutes;

H_1 : Mean Active Repair Time = 60 minutes;

with $\alpha = 0.10$ and $\beta = 0.10$.

The demonstration test for this specification will be such that the probability of rejecting a system whose MART is 30 minutes is 0.10, while the probability of accepting a system whose MART is 60 minutes is 0.10.

1.4 The Maintainability parameter should be specified in the system specification and should be representative of the desired system characteristics when in-service. Obviously the parameter must be a measure which the producer can influence through design. This chapter discusses the sampling and statistical evaluation procedures required to demonstrate conformance to the requirement. Leaflet D11/1 describes the various test methods available.

2 CONCEPTS

2.1 Hypothesis Testing

2.1.1 The procedure for hypothesis testing is to establish the appropriate hypothesis and its alternative before the demonstration is conducted. Then the hypothesis can be tested with the appropriate statistics determined from the sample data.

2.1.2 The first step is to set up the null hypothesis H_0, that is there is no real change or difference between the sample and the population, and to test the null hypothesis against an alternative hypothesis, of which there are many alternatives. For example, suppose the required Mean Active Repair Time (MART) for a system is μ_0 (population mean). We test a sample of 30 repair tasks to obtain an observed mean μ. The null hypothesis is that the mean of the sample equals the MART requirement. The alternative hypothesis H_1 is that the sample mean is greater than μ_0:

Null hypothesis $H_0 : \mu = \mu_0$

Alternative hypothesis $H_1 : \mu > \mu_0$

2.2 Producer's and Consumer's Risks

2.2.1 One cannot expect the sample mean to equal exactly the expected population mean. Therefore, we must allow for variation between the means. The variation is described by two types of errors:

a) Type I error - the test concludes that the sample mean was not equal to the requirement (population), when in fact $\mu = \mu_0$. The probability of making a type I error is α (producer's risk).

b) Type II error - the test concludes that the sample mean was equal to the requirement (population) when in fact $\mu \neq \mu_0$. The probability of making a type II error is β (consumer's risk).

These errors can be summarised by the table below:

Decision	H_0 is true	H_0 is false
Accept H_0	No error	Type II error
Reject H_0	Type I error	No error

Table 1: Summary of Errors Associated with Hypothesis Testing

2.2.2 Figure 1 is a visual presentation of the Type I and Type II errors. For a given decision point, Type I error (α) is part of the population distribution below the decision point. These are test results which belong to the population distribution, with mean μ_0, but would be rejected. Type II error (β) is part of the sample distribution above the decision point. It is evident from Figure 1, that changing the producer's (α) or consumer's (β) risk will change the decision point. For a fixed sample size, as β decreases, α increases.

Figure 1: Type 1 and Type II Errors

2.2.3 The values for the producer's and consumer's risk are sometimes given in the requirements specification. However, often the onus is put on the contractor to develop a plan which will be acceptable to the customer. The implications as to whether to minimise type I or type II errors needs to be considered. For example, in the example described in 2.1.2, a type I error to reject H_0 when, in fact, H_0 is true would mean the failure of the Maintainability Demonstration, the possibility of re-designs leading to programme delay and financial loss to the contractor. On the other hand, a type II error, accepting H_0 when it is false, would mean that the system maintainability was not as good as required, leading to higher support costs in-service. In most engineering situations, a type II error is least desirable and should be minimised.

3 TEST METHODS

3.1 Test Parameters

3.1.1 Each test method has a Maintainability parameter which it is designed to demonstrate, assumptions and the required sample size and selection method. Table 2 presents a summary of the test methods available for planning a demonstration. The choice of test method will depend on a number of factors, including the Maintainability parameter, and any statistical assumptions related to the maintainability parameter of interest.

Test Plan	Test Parameter	Assumptions	Sample Size	Sample Selection Method
MIL-HDBK-470A Test 1 - A & BS 6548 : Part 6 Test 1	Mean	Log-normal distribution and prior knowledge of variance	No minimum in MIL-HDBK-470A, 30 in BS 6548	Natural occurring failures or stratification in MIL-HDBK-470A, or simple random sampling in BS 6548

Table 2: Summary of Available Maintainability Demonstration Test Methods

(part 1 of 2)

Test Plan	Test Parameter	Assumptions	Sample Size	Sample Selection Method
MIL-HDBK-470A Test 1 - B	Mean	No distribution assumption, prior knowledge of variance	No minimum	Natural occurring failures or stratification
MIL-HDBK-470A Test 2 & BS 6548 : Part 6 Test 4	Critical Percentile (Fractile)	Log-normal distribution and prior knowledge of variance	No minimum in MIL-HDBK-470A, 20 in BS 6548	Natural occurring failures or stratification in MIL-HDBK-470A, or simple random sampling in BS 6548

Test Plan	Test Parameter	Assumptions	Sample Size	Sample Selection Method
MIL-HDBK-470A Test 3	Critical Maintenance Time or Manhours	None	No minimum	Natural occurring failures or stratification
MIL-HDBK-470A Test 4	Median	Log-normal distribution	Must be 20	Natural occurring failures or stratification
MIL-HDBK-470A Test 5	Chargeable Maintenance Downtime/ Flight	Only valid if the Central Limit Theory applies. No assumption regarding probability distribution	Minimum of 50	Natural occurring failures
MIL-HDBK-470A Test 6	Manhour Rate	Non-statistical test	No minimum	Natural occurring failures
MIL-HDBK-470A Test 7	Manhour Rate	None	Minimum of 30	Natural occurring failures or stratification
MIL-HDBK-470A Test 8	Mean and Percentile/ Dual Percentile	Log-normal distribution. See also Leaflet 6/1.	Sequential test	Natural occurring failures or simple random sampling
MIL-HDBK-470A Test 9	Mean (Corrective Task Time, Preventive Maintenance Time, Downtime)/ Mmax (90 or 95 Percentile)	Only valid if the Central Limit Theory applies. No assumption regarding probability distribution	Minimum of 30	Natural occurring failures or stratification
MIL-HDBK-470A Test 10	Median (Corrective Task(ct) Time, Preventive Maintenance(pm) Time), Mmaxct (95 Percentile), Mmaxpm (95 Percentile)	None	Minimum of 50	Natural occurring failures or stratification
MIL-HDBK-470A Test 11	Mean (Preventive Maintenance Task Time). Mmax (Preventive Maintenance Task Time at any Percentile)	None	None	All
BS 6548 : Part 6 Test 2	Mean	No distribution assumption, prior knowledge of variance	Minimum of 30	Simple random sampling
BS 6548 : Part 6 Test 3	Mean	None	Minimum of 50	Simple random sampling
BS 6548 : Part 6 Test 5	Proportion of Corrective Task Times above a specified value	Log-normal distribution	Minimum of 20	Simple random sampling
BS 6548 : Part 6 Test 6	Proportion of Corrective Task Times above a specified value	None	No minimum	Simple random sampling
BS 6548 : Part 6 Test 7	Proportion of Corrective Task Times above a specified value	None	Sequential test	Simple random sampling
NES 1017/MDEM	Details not available at the time of writing			

Table 2: Summary of Available Maintainability Demonstration Test Methods (part 2 of 2)

3.1.2 The mission profile of the equipment is often the main criterion for selecting a particular Maintainability parameter. If the equipment is mission critical, then equipment downtime will determine the Maintainability parameter to be demonstrated. However, if the equipment is not mission critical, then manpower may be the more important characteristic. Often emphasis is placed on corrective maintenance as this is unscheduled and could result in an interruption to the mission; whereas preventive maintenance can be scheduled during periods of non-use. However, for equipment in continuous use, then the total maintenance time is important. For one shot devices, such as a missile system, corrective and preventive maintenance must be considered separately.

3.1.3 If the requirement for a system is either operational or intrinsic Availability, given that:

$$\text{Operational Availability} = \frac{\text{MTBM}}{\text{MTBM} + \text{MDT}}$$

Where MTBM = Mean Time Between Maintenance

MDT = Mean Down Time

and

$$\text{Intrinsic Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

Where MTBF = Mean Time Between Failure

MTTR = Mean Time Between Repair

then the maintainability parameter to be demonstrated would be MDT and MTTR respectively.

3.1.4 A Maintenance Free Operating Period (MFOP) is a requirement which can be applied to military platform and is defined as a period where the platform is maintenance free. However, the MFOP will be followed by a Maintenance Recovery Period (MRP), which is the downtime during which the appropriate preventive or corrective maintenance is done to recover the system to its fully serviceable state so that it is capable of achieving the next MFOP. For this type of requirement, a maximum downtime or 0.95 probability of completing the maintenance within a specific time would be the most appropriate Maintainability parameter. It is important to note that the Maintainability parameter of interest may vary depending on the maintenance level. Typically, maintenance levels nearer the operational front line will tend to have Maintainability parameters which define equipment downtime, whereas more remote levels will have parameters which define maintenance manhours.

3.2 Choosing a Test Method

3.2.1 The test method to be used to demonstrate Maintainability is often determined by the Maintainability parameter of interest. However, if prior knowledge exists, perhaps from a Maintainability estimation, then a sequential test can result in a significant reduction in sample size, otherwise a fixed sample size test would be required. The benefit of a fixed sample size is that the number of tasks is known prior to the demonstration which is therefore easier to plan.

3.2.2 Analysis has shown that in many situations a log-normal distribution provides a good estimation of corrective maintenance repair times. However, it is not safe to assume that every system will have repair times which are log-normally distributed. For equipment with a large amount of electronics or a high degree of built-in diagnostics, the distribution should be tested through use of goodness-of-fit tests such as Chi-square or Kolomogorov-Simirnov (see PtDCh7).

4 TASK SELECTION METHODS

Table 3: Calculations of Relative Frequency and Sample Size for an Example Coolant System

4.1 General

4.1.1 Task selection methods are only required when failure simulation is used to generate maintenance tasks, rather than for naturally occurring failures. The two widely used methods are:

a) Stratified sampling

b) Non-stratified sampling

The object of both methods is to determine a hypothetical task population which will be representative of the total population of the tasks for the system.

4.2 Non-Stratified Sampling

4.2.1 This sampling method enables representative tasks to be selected based on the relative frequency of task occurrence and is probably the most commonly used approach in the UK. Table 3 shows the computations used on a example coolant system. The following presents a step by step approach based on NES 1017 and MIL-HDBK-470A:

a) Identify the major items which make up each equipment within the whole equipment.

b) Subdivide each major item to a unit level at which maintenance will be performed, as defined in the maintenance plan.

c) Identify the maintenance tasks associated with each unit defined in b. Note that these are tasks and not failure modes, and the same task may be required for different failure modes of the same unit.

d) From the failure rate prediction (see PtCCh36) assign a failure rate for each unit maintenance task as identified in b.

e) Determine the number of units in each major item

f) Determine the resulting failure rate for each unit. This is the product of the unit failure rate from d., the number of units from e. and their duty cycle.

g) Sum the failure rates for the equipment identified in f.

h) Determine the relative frequency of each unit, by dividing the failure of each unit by that of the whole equipment.

i) For a Fixed Sample Test Method, the number of tasks per unit level task can then be calculated by multiplying the relative frequency determined in h. by the sample size specified by the selected test.

a) For a Sequential Test Method, the relative frequencies of each unit level task are added to determine a cumulative frequency range for each unit level task. For example, in Table 2 the first unit range is 0.0 to 0.024, and the next 0.024 to 0.108. Using random numbers, maintenance tasks are selected whose range of frequency includes the random number obtained. This task is demonstrated and the process repeated until an accept/reject decision is reached.

4.2.2 A Failure Mode and Effects Analysis (FMEA) (see PtCCh33), down to a level at which at which maintenance is to be performed, could be utilised for this task selection method. The FMEA will provide the failure modes, which result in the maintenance tasks for consideration, and the failure rates for the relative frequency of tasks.

4.3 Stratified Random Sampling

4.3.1 The object of stratification is to divide the system into groups with similar task characteristics. The maintenance tasks within each group should be of a similar task type (e.g. remove and replace, clean and grease etc.), have similar task characteristics (e.g. significant diagnosis time, or short setting to work time, etc.), and similar total repair times. Stratification ensures that the tasks which are selected are not biased towards one task type, characteristic or repair time.

4.3.2 Engineering judgement plays an important part in stratification, as two very different maintenance tasks, such as replacing a PEC board and renewing a mechanical valve may have similar repair times, but it would inappropriate to group the two tasks. The stratification process based on MIL-HDBK-470A is illustrated in Table 3 and is summarised by step by step approach below.

a) Steps a., b. and c. are as presented in Section 4.2.1 above.

b) Identify for each maintenance task the estimated maintenance time from the maintainability estimation (See PtCCh37).

c) Steps d., e. and f. are as in Section 4.2.1 above

d) Group maintenance tasks which have similar characteristics, such as similar maintenance actions and estimated maintenance times.

e) For each grouping, determine the total failure rate by summing the product of the unit failure rate, and the number of units for all tasks within the group.

f) Determine the relative frequency of occurrence for each task grouping by dividing the sum of the total failure rate into the individual total failure rate for each group.

g) For a Fixed Sample Test Method a sample of maintenance tasks equal to four times the sample sized defined by the test method is allocated to the task groups appropriate to its relative frequency of occurrence. (i.e. for a test method requiring a minimum sample size of 30, the population sample size would be 4 x 30 =120)

h) Allocate the maintenance tasks among the task groups in accordance with the relative frequency of occurrence of maintenance group. The task which is actually demonstrated, is then selected from the maintenance tasks allocated to the group. Note: Once the maintenance task has been demonstrated it is not returned to the sample and therefore is only demonstrated once.

i) For a Sequential Test Method, the relative frequency of each unit level task is added to the previous, in order to determine a cumulative frequency range for each unit level task. Using random numbers, maintenance tasks are selected whose range of frequency includes the random number obtained. This task is demonstrated and the process repeated until an accept/reject decision is reached. The demonstrated task is then returned to the sample.

Table 4: Stratification Process for an Example Coolant System

(Part 1 of 2)

Table 4: Stratification Process for an Example Coolant System

(Part 2 of 2)

4.4 Maintenance Task Selection

4.4.1 The methods listed above describe how to determine the sample sizes, but not how to select the sample tasks for demonstration. NES 1017 states that for each item requiring a demonstrated task, a FMEA should be consulted to determine the predominant failure mode to be simulated in the item. Otherwise, if a number of failure modes are possible, to employ a further simple random sampling method to determine which failure mode to use.

4.4.2 MIL-HDBK-470A suggests a similar method based on the frequency of occurrence of failure modes. Table 3 indicates the allocation of maintenance tasks for each group of similar tasks, based on their estimated frequency of occurrence. The population allocation for the Thermostatic Control Valve (TCV) is three, which means that at least three failure modes must be considered, from which only one will be selected for simulation. To select the failure mode for simulation, a random sampling procedure is used based on the relative frequency of occurrence of the failure modes.

4.4.3 Even when the failure mode to be simulated has been chosen, there will still be different ways of inducing the failure. Some methods of failure inducement will result in different symptoms, which may be either easier or more difficult to detect. This will not actually affect the maintenance action which takes place, but there is a possibility that the maintenance time will be affected.

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REFERENCES

1. MIL-HDBK-470A. Volume 1. 04 August 1997. *Designing and Developing Maintainable Products and Systems.* Volume 1. US Department of Defense.

2. BS6548 : Part 6: 1995. *Maintainability of Equipment. Part 6. Guide to Statistical Methods in Maintainability Evaluation.* BSI Standards.

3. DGSS/ADPSS/26/5/7. Issue 3. June 1996. *The Maintainability Demonstration (MDEM) Model Guide to PC Operators.* MoD(PE).

4. NES 1017. Issue 3. January 1993. *Requirements for Maintainability Demonstrations of Naval Systems.* MoD(PE).

RELATED DOCUMENTS

1. DEF-STAN 00-40. *Reliability and Maintainability.* MOD(PE).

2. *Maintenance Free Operating Periods - The Designs Challenge.* M N Relf. Paper presented to 13th Advances in Reliability Technology Symposium (April 1998). BAe.

3. *Delivering Failure Free Operations.* C Hockley. Paper presented to 12 Technical Proceedings of the SRD (June 1997).

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MAINTAINABILITY DEMONSTRATION TEST METHODS

CONTENTS

5 INTRODUCTION AND ACKNOWLEDGEMENT

5.1 This leaflet duplicates Sections B4.1 to B4.12 of Appendix B to MIL-HDBK-470A. This reference is a comprehensive description of the most frequently used maintainability demonstration test methods, and includes many examples.

6 EXTRACT FROM MIL-HDBK-470A, APPENDIX B

6.1.1.1.1 **List of Symbols.** The following symbols and notations are common to test methods 1 – 3 contained in this appendix.

X = the random variable which denotes the maintenance characteristics of interest (e.g. X can denote corrective maintenance time, preventative maintenance time, fault location time, man-hours per maintenance task, etc.).

X_i = the i^{th} observation or value of the random variable X.

n = the sample size.

$\overline{X}$ = the sample mean $\overline{X} = \frac{1}{n}\sum_{i=1}^{n}(X_i))$

E(random variable) = the expected value of the variable

σ^2 = $E[(\ln X - \theta)^2]$ = the true variance of lnX

μ = E(X) = the true mean of X

$\hat{d}^2$ = $Var(X) = E[(X-\mu)^2]$ = the true variance of X

$\hat{d}^2$ = the sample variance of X (i.e. $\hat{d}^2$ =

$$\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}X_i^2 - n\overline{X}^2\right)$$

$\tilde{d}^2$ = the prior estimate of teh variance of the maintenance time.

X_p = the (1-p)th percentile of X (i.e. $X_{.05}$ = the 95^{th} percentile of X).

$\tilde{M}$ = $X_{.05}$ = the median of X.

Y = ln(X) = the natural logarithm of X.

$\overline{Y}$ = the sample mean of Y.

θ = E(ln(X)) = the true mean of ln(X).

$\tilde{\sigma}^2$ = the prior estimate of the variance of the logarithm of maintenance times.

s^2 = the sample variance of ln(X).

Z_p = the standardised normal deviate exceeded with probability p (i.e.

$$\int_{Z_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-z^2}{2}\right)} dz = p)$$

Z_α = $Z_{(1-\beta)}$ = standardised normal deviate exceeded with probabilities α and $(1-\beta)$ respectively.

α = the producer's risk: the probability that the equipment will be rejected when it has a true value equal to the desired value (H_0).

β = the consumer's risk: the probability that the equipment will be accepted when it has a true value equal to the maximum tolerable value (H_1).

H_0 = the desired value specified in the contract or specification and is expressed as a mean, critical percentile, or critical maintenance time.

H_1 = the maximum tolerable value. Note: $H_0 < H_1$.

When X is a log-normally distributed random variable:

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} e^{-1/2\sigma^2} (1nx - \theta)^2, 0 < x < \infty$$

If Y = ln(X) the probability density of Y is normal with mean θ and σ^2 variance

$Y \sim N(\theta.\sigma^2)$

Properties of the log-normal distribution:

$$\text{mean} = \mu = e^{\left(\theta + \frac{\sigma^2}{2}\right)}$$

$$\text{variance} = d^2 = e^{\left(\theta+\sigma^2\right)}\left(e^{\sigma^2} - 1\right)$$

$$\text{median} = \tilde{M} = e^{\vartheta}$$

$$\text{mode} = M = e^{\left(\theta-\sigma^2\right)}$$

$$\text{(1-p)th percentile} = X_p = e^{\left(\theta + Zp\sigma\right)}$$

TABLE B_VIII. Standardised Normal Deviates

P	**Z_p**
0.01	2.33
0.05	1.65
0.10	1.28
0.15	1.04
0.20	0.84
0.30	0.52

The following symbols are common to test methods 4, 8-11 contained in this appendix.

X_{ci} = Maintenance downtime per corrective maintenance task (of the i^{th} task).

X_{pm_j} = Maintenance downtime per preventative maintenance task (of the i^{th} task).

n_C = Number of corrective maintenance tasks sampled.

n_{pm} = Number of preventative maintenance tasks sampled.

β = Consumer's risk.

ϕ = That value, corresponding to risk, which is obtained from a table of normal distribution for a one-tail test.

f_c = Number of expected corrective maintenance tasks occurring during a representative operating time (T).

f_{pm} = Number of expected preventive maintenance tasks occurring during a representative operating time (T).

T = Item representative operating time period.

D_t = Total maintenance downtime in the representative operating time (T).

$\overline{X}_c, \overline{X}_{pm}, \overline{X}_{p/c}$ = Mean downtimes of sample. (Corrective, Preventive and combined Corrective/Preventive Maintenance Times).

M'_{Max_C} = Sample calculated maximum corrective maintenance downtime.

μ_c = Specified mean corrective maintenance time.

μ_{pm} = Specified mean preventive maintenance time.

$\mu_{p/c}$ = Specified mean maintenance time. (Taking both corrective and preventive maintenance time into account)

M_{Max} = A requirement levied in terms of a maximum value of a percentile of task time (i.e. 95% of all corrective task times must be less than 60 minutes) usually taken as the 90th or 95th percentile.

$M_{Max\,c}$ = Specified M_{Max} of corrective maintenance downtimes.

$M_{Max\,pm}$ = Specified M_{Max} of preventive maintenance downtimes.

θ_c = $E(\ln X_c)$ = Expected value of the logarithm of corrective maintenance tasks.

$\text{Log}\, X_{ci} . \text{Log}\, X_c$ = Log to the base 10 of X_{ci}, X_c

$\ln X_{c_i} . \ln X_c$ = Natural logs of X_{ci}, X_c

$\tilde{M}_{ct}$ = Median value of corrective maintenance tasks.

$\tilde{M}_{pm}$ = Median value of preventive maintenance tasks.

6.1.1.1.2 **TEST METHOD 1: Test On The Mean.** This test provides for the demonstration of maintainability when the requirement is stated in terms of both a required mean value (μ_1) and a design goal value (μ_0) (or when the requirement is stated in terms of a required mean value (μ_1) and a design goal value (μ_0) is chosen by the contractor). The test plan is subdivided into two basic procedures identified herein as Test Plan A and Test Plan B. Test A makes use of the lognormal assumption for determining the sample size, whereas Test B does not. Both tests are fixed sample tests (minimum sample size of 30), which employ the Central Limit Theorem and the asymptotic normality of the sample mean for their development.

ASSUMPTIONS

Test A - Maintenance times can be adequately described by a lognormal distribution. The variance, σ^2, of the logarithms of the maintenance times is known from prior information or reasonable precise estimates can be obtained.

Test B - No specific assumption concerning the distribution of maintenance times are necessary. The variance d^2 of the maintenance times is known from prior information or reasonably precise estimates can be obtained.

Hypotheses

$$H_0 : \text{Mean} = \mu_0 \qquad \text{(Equation B-2)}$$

$$H_1 : \text{Mean} = \mu_1 . (\mu_1 > \mu_0) \qquad \text{(Equation B-3)}$$

Illustration: $H_0 : \mu_0 = 30\ \text{minutes}$

$$H_1 : \mu_1 = 45 \text{ minutes}$$

Note that μ_0 is normally the specified maintainability index value, and that μ_1 is typically the maximum acceptable value of the specified index.

<u>SAMPLE SIZE</u> - For a test with producer's risk α and consumer's risk (β) the sample size for Test A is given by:

$$n = \frac{(Z_\alpha \mu_0 + Z_\beta \mu_1)^2}{(\mu_1 - \mu_0)^2} (e^{\tilde{\sigma}^2} - 1) \qquad \text{(Equation B-4)}$$

where $\tilde{\sigma}^2$ is a prior estimate of the variance of the maintenance times and Z_α and Z_β are standardised normal deviates. The sample size for Test B is given by:

$$n = \left(\frac{Z_\alpha + Z_\beta}{\frac{\mu_1 - \mu_0}{\tilde{d}}} \right)^2 \qquad \text{(Equation B-5)}$$

where $\tilde{d}^2$ is a prior estimate of the variance of the maintenance times. Z_α and Z_β are standardised normal deviates.

<u>Decision Procedure</u> - Obtain a random sample of n maintenance times, X_1, X_2 …… X_n and compute the sample mean.

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad \text{(Equation B-6)}$$

and the sample variance

$$\hat{d}^2 = \frac{1}{n-1} \left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2 \right) \qquad \text{(Equation B-7)}$$

<u>Test A:</u> Accept if $\overline{X} \le \mu_0 + Z_\alpha \dfrac{\hat{d}}{\sqrt{n}}$ (Equation B-8)

<u>Test B:</u> Accept if $\overline{X} \le \mu_0 + Z_\alpha \dfrac{\hat{d}}{\sqrt{n}}$ (Equation B-9)

Reject otherwise.

<u>Discussion</u> - By the central limit theorem, the sample mean $\overline{X}$ is appropriately normal for large n with mean E(X) and variance Var $(\overline{X})$. In Test A, under the log-normal assumption Var $\overline{X} = d^2$ where $d^2 = e^{\left(2\theta + \tilde{\sigma}^2\right)} (e^{\tilde{\sigma}^2} - 1) = \mu^2 (e^{\tilde{\sigma}^2} - 1)$. Thus the sample size n, can be

computed using a prior estimate of $\tilde{\sigma}^2$. In Test B, a prior estimate of d^2 is assumed to be available to calculate the sample size. A critical value C is chosen such that $\mu_0 + Z_\alpha \sqrt{\mathrm{Var}\overline{\overline{X}}} = C = \mu_1 - Z_\beta \sqrt{\mathrm{Var}\overline{\overline{X}}}$.

If $\mu = \mu_0$, then $P(\overline{X} > C) = \alpha$ and if $\mu = \mu_1$ then $P(\overline{X} \le C) = \beta$.

Example – It is desired to test the hypothesis that the mean corrective maintenance time is equal to 30 minutes against the alternate hypothesis that the mean is 45 minutes with $\alpha = \beta = 0.05$.

Then $H_0 : \mu_0 = 30$ minutes

$H_1 : \mu_1 = 45$ minutes

Test A: Under the log-normal assumption with prior estimate of $\tilde{\sigma}^2 = 0.6$, the sample size using equation B-4 is: $n_c = \frac{[1.65(30) + 1.65(45)]^2}{(45-30)^2}(e^{0.6} - 1) = 56.$

Test B: Under the distribution-free case with a prior estimate of $\tilde{d}^2 = 900.$ (or $\tilde{d} = 30$) the sample size using equation B-5 is:

$$n_c = \left[\frac{3.29}{\left(\frac{45-30}{30}\right)}\right]^2 = 43$$

Operating Characteristic (OC) Curve – The OC curve for Test B for this example is given in Figure B-4. It gives the probability of acceptance for values of the mean maintenance time from 20 to 60 minutes. The OC curve for Test A for this example is given in Figure B-3. It gives the probability of acceptance for various values of the mean maintenance time. Thus, if the true value of μ is 40 minutes, then the probability that a demonstration will end in acceptance is 0.21 as seen from Figure B-3.

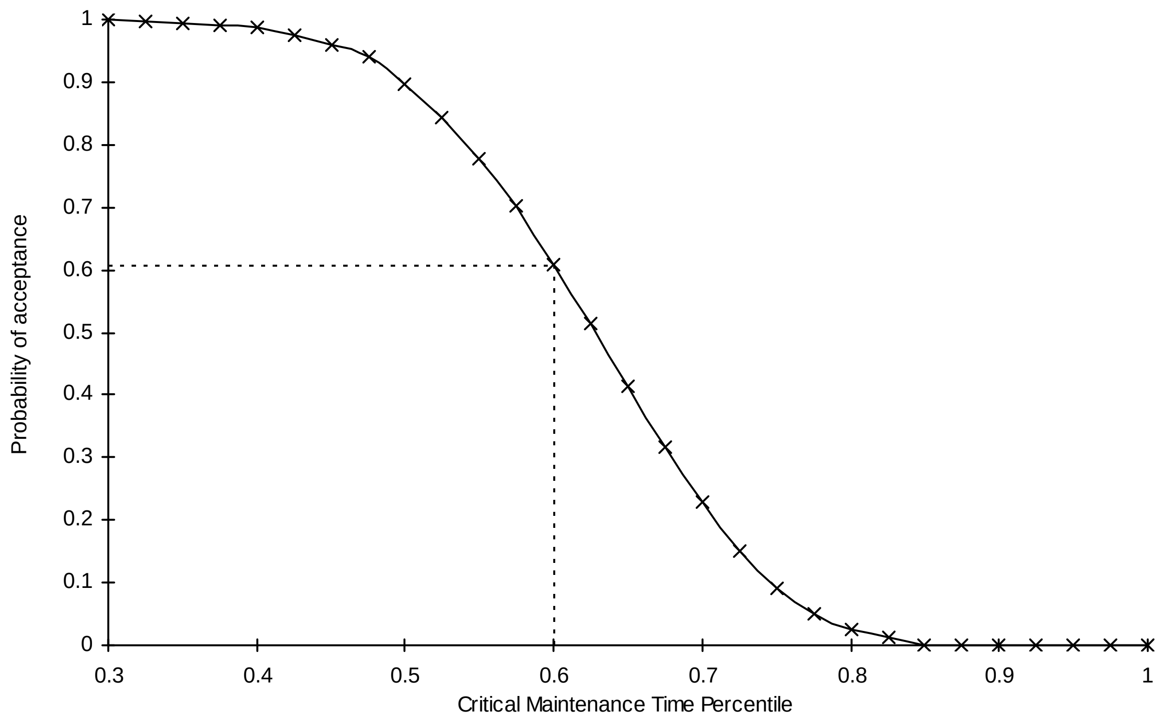

Figure B-3. OC Curve for Test A

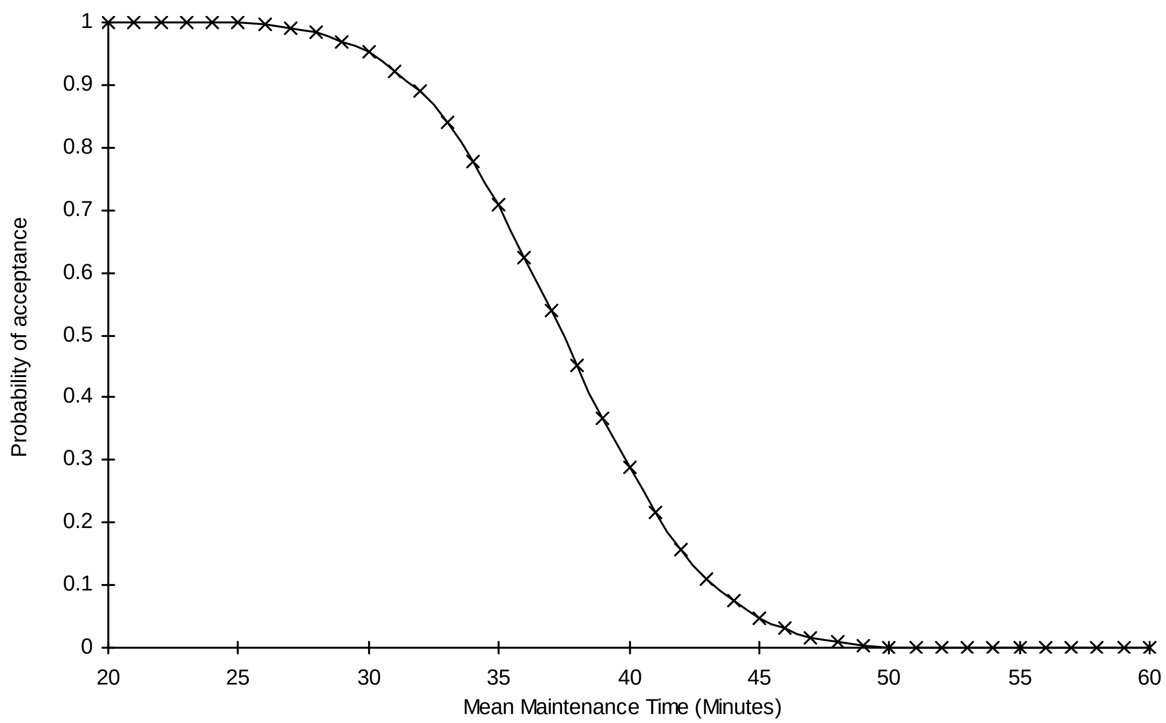

Figure B-4. OC Curve for Test B

6.1.1.1.3 **TEST METHOD 2: Test On Critical Percentile.** This test provides for the demonstration of maintainability when the requirement is stated in terms of both a required critical percentile value (T_1) and a design goal value (T_0) [or when the requirement is stated in terms of a required percentile value (T_1) and a design goal value (T_0) is chosen by the system developer]. If the critical percentile is set at 50 percent, then this test method is a test of the median. The test is a fixed sample size test. The decision criterion is based upon the asymptotic normality of the maximum likelihood estimate of the percentile value.

ASSUMPTIONS

Maintenance times can be adequately described by a log-normal distribution. The variance, σ^2, of the logarithms of the maintenance times is known from prior information or reasonably precise estimates can be obtained.

HYPOTHESES

H_0 : (1-p)th percentile, $X_p = T_0$ (Equation B-10)

or $P[X > T_0] = p$

H_1 : (1-p)th percentile, $X_p = T_1$ (Equation B-11)

or $P[X > T_1] = p$, $(T_1 > T_0)$

Illustration: H_0 : 95th percentile = $X_p = X_{.05} = T_0 = 1.5$ hours

$\ln T_0 = 0.4055$

H_1 : 95th percentile = $X_p = X_{.05} = T_1 = 2$ hours

$\ln T_1 = 0.6932$

SAMPLE SIZE - To meet specified α and β risks, the sample size to be used is given by the formula

$$n = \left(\frac{2 + Z_p^{\,2}}{2}\right)\tilde{\sigma}^2 \left(\frac{Z_\alpha + Z_\beta}{\ln T_1 - \ln T_0}\right)^2 \quad \text{(Round up to next integer)} \qquad \text{(Equation B-12)}$$

where:

$\tilde{\sigma}^2$ is a prior estimate of σ^2, the true variance of the logarithms of the maintenance times.

Z_p is the standardised normal deviate corresponding to the (1-p)th percentile.

DECISION PROCEDURE - Compute:

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} \ln X_i \qquad \text{(Equation B-13)}$$

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} (\ln X_i)^2 - n\overline{Y}^2\right] \qquad \text{(Equation B-14)}$$

$$X^* = \ln T_0 + Z_\alpha s\left[\frac{1}{n} + \frac{Z_p^{\ 2}}{2(n-1)}\right]^{1/2} \qquad \text{(Equation B-15)}$$

$$\text{Accept if } \overline{Y} + Z_p s \le X^* \qquad \text{(Equation B-16)}$$

Reject otherwise.

Discussion - This test is based upon the fact that under the log-normal assumption, the (1-p)th percentile value is given by $X_p = e^{(\theta + Z_p\sigma)}$. Taking logarithms gives ln $X_p = \theta + Z_p\sigma$, and using maximum likelihood estimates for the normal parameters θ and σ, the (1-p)th percentile maximum likelihood estimate is $\ln \hat{X}_p = \overline{Y} + Z_p\sigma\sqrt{\frac{n-1}{n}}$. ln X_p is approximately normal. To meet the producer's risk requirements, a critical value X* is chosen for the sample estimate of the (1-p)th percentile X_p. Note $\overline{Y} = \hat{\theta}$ is an estimate for θ.

Example - The following hypotheses are to be tested at $\alpha = \beta = .10$

H_0 : 95th percentile = $X_{.05}$ = 1.5 hours = T_0; ln T_0 = .4055

H_1 : 95th percentile = $X_{.05}$ = 2.0 hours = T_1; ln T_1 = .6932.

A prior estimate of $\tilde{\sigma}^2$ is equal to 1.0. Using equation B-12,

$$n_c = \left(\frac{2 + (1.65)^2}{2}\right)1.0\frac{(2.65)^2}{(\ln 2.0 - \ln 1.5)^2}$$

The critical value X* is given by equation B-15,

$$X^* = \ln T_0 + Z_\alpha s\left[\frac{1}{n} + \frac{Z_p^{\ 2}}{2(n-1)}\right]^{1/2}$$

$$= \ln 1.5 + 1.28s\left[\frac{1}{187} + \frac{(1.65)^2}{372}\right]^{1/2}$$

$$= .4055 + .1437s$$

OC Curve - The OC curve for Test Method 2 for this example is given in Figure B-5. It gives the probability of acceptance for various values of the 95th percentile of the maintenance time distribution. If the true value of $X_{0.05}$ is 1.7 hours, then the probability that a demonstration will end in acceptance is 0.57 as seen from Figure B-5.

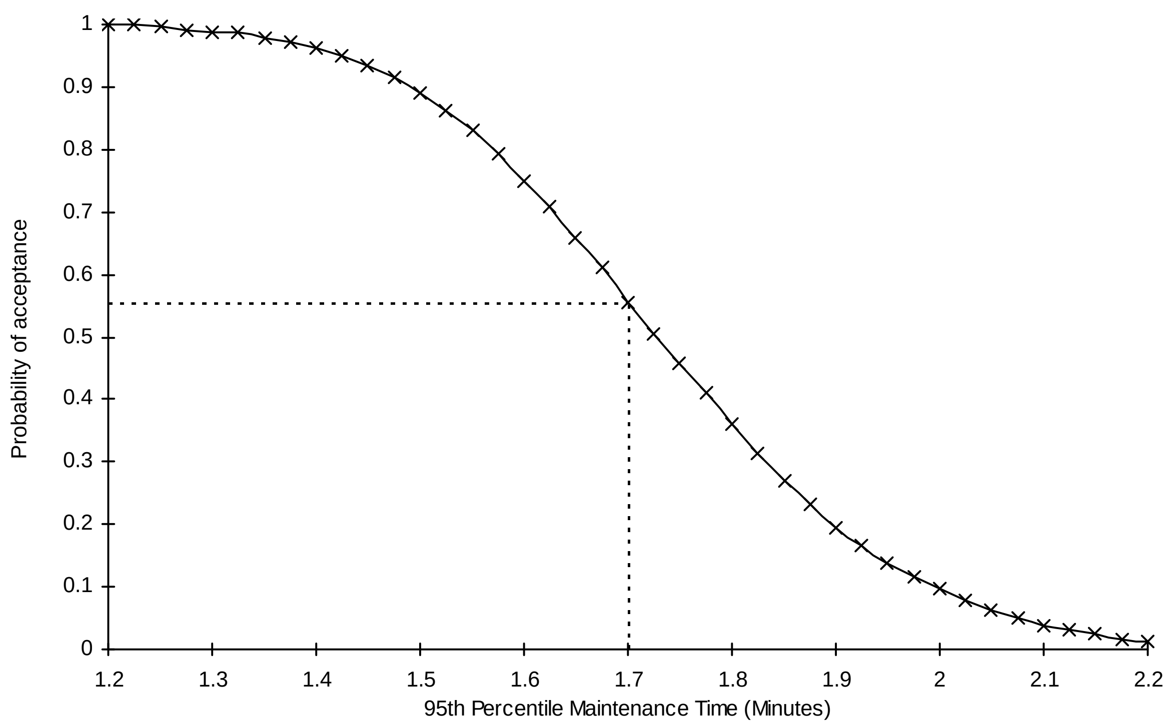

Figure B-5. OC Curve for Test Method 2

6.1.1.1.4 **TEST METHOD 3: Test On Critical Maintenance Time or Manhours.** This test provides for the demonstration of maintainability when the requirement is specified in terms of both a required critical maintenance time (or critical manhours) (X_{P1}) and a design goal value (X_{P0}) (or when the requirement is stated in terms of a required critical maintenance time (X_{P1}) and a design goal value (X_{P0}) is chosen by the system developer). The test is distribution-free and is applicable when it is desired to establish controls on a critical upper value on the time or manhours to perform specific maintenance tasks. In this test both the null and alternate hypothesis refer to a fixed time and the percentile varies. It is different from Test Method 2 where the percentile value remains fixed and the time varies.

ASSUMPTIONS - No specific assumption is necessary concerning the distribution of maintenance time or manhours.

HYPOTHESES

$H_0 : T = X_{P0}$

$(P_1 > P_0)$ (Equation B-17)

$H_1 : T = X_{P1}$ (Equation B-18)

For a specified α and β.

Illustration

H_0 : 30 minutes = $X_{0.50}$ = 50th percentile (median)

H_1 : 30 minutes = $X_{0.75}$ = 25th percentile.

SAMPLE SIZE, n, AND ACCEPTANCE NUMBER, c -

The normal approximation to the binomial distribution is employed to find n and c when P_0 is not a small value. Otherwise, the Poisson approximation is employed. The equations for n and c are as follows:

For $0.20 < p_0 < 0.80$ ($p_i = 1 - Q_i$)

$$n = \left[\frac{Z_\beta \sqrt{p_1 Q_1} + Z_\alpha \sqrt{p_0 Q_0}}{p_1 - p_0} \right]^2$$ (Use next higher integer value) (Equation B-19)

$$c = n \left[\frac{Z_\beta p_0 \sqrt{p_1 Q_1} + Z_\alpha p_1 \sqrt{p_0 Q_0}}{Z_\alpha \sqrt{p_0 Q_0} + Z_\beta \sqrt{p_1 Q_1}} \right]$$ (Use next lower integer value) (Equation B-20)

For $P_0 < 0.20$, n and c can be found from the following two equations:

$$\sum_{r=0}^{c} \frac{e^{-np0} (np_0)^r}{r!} \geq 1 - \alpha$$ (Equation B-21)

$$\sum_{r=0}^{c} \frac{e^{-np1} (np_1)^r}{r!} \leq \beta$$ (Equation B-22)

Table B-IX provides sampling plans for various α and β risks and ratios p_1 / p_0 when $p_0 < 0.20$.

Decision Procedure - Random samples of maintenance times are taken, yielding n observations X_1, X_2, ... X_n. The number of such observations exceeding the specified time T is counted. This number is called r.

Accept H_0 if $r \leq c$ (Equation B-23)

Reject H_0 if $r > c$ (Equation B-24)

Example - A median value of 30 minutes is considered acceptable whereas if 30 minutes is the 25th percentile then this is considered unacceptable. The following hypotheses result:

H_0 : 30 minutes = $X_{0.50}$ = 50th percentile (median)

H_1 : 30 minutes = $X_{0.75}$ = 25th percentile.

$\alpha = \beta = .10$

Then, $Z_\alpha = Z_\beta = 1.28$, $p_0 = 0.50$, $p_1 = 0.75$. Using equations B-19 and B-20:

$$n = (1.28)^2 \left[\frac{\sqrt{(.75)(.25)} + \sqrt{(.50)(.50)}}{(.25)} \right]^2 \approx 23$$

Table B-IX. Sampling Plans for Specified p_0, p_1, α and β when p_0 is Small (e.g. p_0<0.20)

$k = \frac{p_1}{p_0}$							α = 0.10						α = 0.20					
	β = 0.05		β = 0.10		β = 0.20		β = 0.05		β = 0.10		β = 0.20		β = 0.05		β = 0.10		β = 0.20	
	c	D	c	D	c	D	c	D	c	D	c	D	c	D	c	D	c	D
1.5	66	54.1	54	43.4	39	30.2	51	43	40	33	29	23.2	36	31.8	27	23.5	17	14.4
2	22	15.7	18	12.4	14	9.25	17	12.8	14	10.3	10	7.02	12	9.91	9	7.29	6	4.73
2.5	13	8.46	10	6.17	8	4.7	10	7.02	8	5.43	6	3.9	7	5.58	5	3.84	3	2.3
3	9	5.43	7	3.98	6	3.29	7	4.66	5	3.15	4	2.43	4	3.09	3	2.3	2	1.54
4	6	3.29	5	2.61	4	1.97	4	2.43	3	1.75	2	1.1	3	2.3	2	1.54	1	0.824
5	4	1.97	3	1.37	3	1.37	3	1.75	2	1.1	2	1.1	2	1.54	1	0.824	1	0.824
10	2	0.818	2	0.818	1	0.353	1	0.532	1	0.532	1	0.532	1	0.824	1	0.824	0	0.227

To find the sample size n, for given p_0, p_1, α and β, divide the appropriate D value by p_0 and use the greatest integer less than the quotient.
Example: p_0 = 0.05, p_1 = 0.20, α = 0.10, β = 0.05. Then k = 0.20/0.05 = 4 and n = D/0.05 = 2.43 / 0.05 = 48 . The acceptance number is c = 4.

and

$$c = 23\left[\frac{1.28(0.5)\sqrt{(.75)(.25)} + 1.28(.75)\sqrt{(.50)(.50)}}{1.28\sqrt{(.50)(.50)} + 1.28\sqrt{(.75)(.25)}}\right] \approx 14$$

OC Curve - The OC curve for Test Method 3 for this example is given in Figure B-6. It gives the probability of acceptance for values of probability p, varying from 0.3 to 1.0. Here X_p is the (1-p)th percentile. Thus, if the true value of the given critical maintenance time is the 40th percentile, i.e., if the value of p is 0.6, then the probability that a demonstration will end in acceptance is 0.61 as seen from Figure B-6.

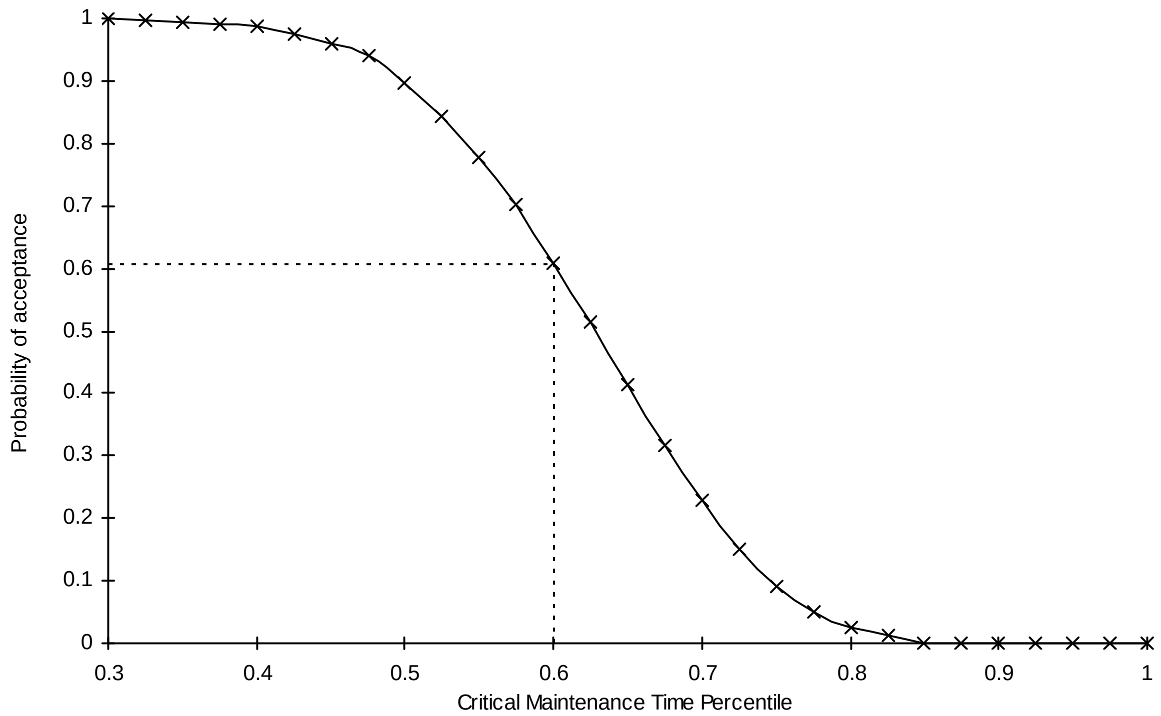

Figure B-6: OC Curve for Test Method 3

6.1.1.1.5 **TEST METHOD 4: Test on the Median (ERT).** This method provides for demonstration of maintainability when the requirement is stated in terms of an Equipment Repair Time (ERT) median, which will be specified in the detailed equipment specification.

ASSUMPTION

This method assumes the underlying distribution of corrective maintenance task times is lognormal.

SAMPLE SIZE

The sample size required is 20. This sample size must be used to employ the equation described in this test method.

TASK SELECTION AND PERFORMANCE - Sample tasks are selected in accordance with the stratification procedure. The duration of each task is recorded and used to compute the following statistics:

$$\text{Log}\,\text{MTTR}_G = \frac{\sum_{i=1}^{n_c} (\text{Log}X_{ci})}{n_c} \qquad \text{(Equation B-25)}$$

$$S = \sqrt{\frac{\sum_{i=1}^{n_c} (\log X_{ci})^2}{n_c} - (\log \text{MTTR}_G)^2} \qquad \text{(Equation B-26)}$$

(Note: All logarithms in equations B-25 and B-26 are to be taken to the base 10.)

Where: $MTTR_G$ is the measured geometric mean time to repair. It is the equivalent to the $\widetilde{M}_{ct}$ used in other plans included in this document.

DECISION PROCEDURE - The equipment under test will be considered to have met the maintainability requirement (ERT) when the measured geometric mean-time-to-repair ($MTTR_G$) and standard deviation (S) as determined in equation B-26 above satisfies the following expression:

Accept if $\log MTTR_G \leq \log ERT + 0.397(S)$ (Equation B-27)

where:

log ERT = logarithm of the equipment repair time

$\log MTTR_G$ = the value determined in accordance with equation B-25

S = the value determined in accordance with equation B-26.

DISCUSSION - The value of equipment repair time (ERT) to be specified in the detailed equipment specification should be determined using the following expression:

ERT (specified) = 0.37 ERT_{max} (Equation B-28)

ERT_{max} = the maximum value of ERT that should be accepted no more than 10 percent of the time.

0.37 = σ value resulting from application of "student's t" operating characteristic that assures a 95 percent probability that an equipment having an acceptable ERT will not be rejected as a result of the maintainability test when the sample size is 20, and assuming a population standard deviation (σ) of 0.55.

DERIVATION OF CRITERIA - The following are brief explanations of the derivations of various criteria specified herein, and are intended for information purposes only. The acceptance criterion, $\log MTTR_G \leq \log ERT + 0.397(S)$, assures a probability of 0.95 of accepting an equipment or system as a result of one test when the true geometric mean-time-to-repair is equal to the specified equipment repair time (that is, a probability of 0.05 of rejecting an equipment or system having a true $MTTR_G$ equal to the specified ERT). This was derived by using conventional methods for establishing acceptance criteria. The conventional methods for determining acceptance based on the measured mean of a small sample (that is, sample size less than 30), and when the true standard deviation (σ) of the population can only be estimated, is to compare the measured mean with the desired mean using the expression:

$$t = \frac{(\bar{x} - \bar{x}_0)}{S}\sqrt{n_c - 1} \qquad \text{(Equation B-29)}$$

where:

$S = \sqrt{\frac{\Sigma_i (x_i - \bar{x})^2}{n_c}}$ or the standard deviation of the sample

$\bar{x}$ = the sample or measured mean

$\bar{x}_0$ = the specified or desired mean

n_c = the sample size

x_i = the value of one measurement of the sample.

The decision to accept the product will be made when the test results give a value of t, as calculated for the above expression, numerically less than or equal to a value of t obtained from "student's t" distribution tables at the established level (that is, 0.99, 0.95, 0.90, etc.) of acceptance and the appropriate sample size. The "student's t" distribution tables (for a single tailed area) give a value to t = 1.729 at the 0.95 acceptance level when the sample size is 20 (that is, 19 degrees of freedom). The table for single tailed area is used since only values of $MTTR_G$ lower than the specified ERT is acceptable. To apply the expression for "t" to the maintainability test, let $\bar{x}_0$ = log ERT (specified), $\bar{x}$ = log $MTTR_G$ (measured), S = the measured standard deviation of the logarithms of the sample of measured repair time, and n_c = the sample size of 20. The measured $MTTR_G$ is then compared to the desired ERT by calculating the value of t using the expression below:

$$t = \frac{(\log MTTR_G - \log ERT)}{S}\sqrt{19}$$

The equipment under test can be acceptable if the value of t calculated from the expression above is equal to or less than +1.729 (the value of t from the "student's t" distribution tables at an acceptable level of 0.95 when the sample size is 20). Therefore, the equipment should be accepted when:

$$\sqrt{19}\,\frac{(\log MTTR_G - \log ERT)}{S} \le +1.729$$

Upon re-arranging and simplifying the above expressionm, the acceptance criterion is obtained as shown below:

$$\log MTTR_G - \log ERT \le \frac{1.729(S)}{\sqrt{19}}$$

$$\log \mathrm{MTTR}_G \leq \log \mathrm{ERT} + 0.397(S)$$ [1]

6.1.1.1.6 **TEST METHOD 5: Test on Chargeable Maintenance Downtime per Flight.** Because of the relatively small size of the demonstration fleet of aircraft and administrative and operational differences between it and fully operational units, operational ready rate or availability cannot be demonstrated directly. However, a contractual requirement for chargeable downtime per flight can be derived analytically from an operational requirement of operational ready rate (ORR) or availability. This chargeable downtime per flight can be thought of as the allowable time (hours) for performing maintenance given that the aircraft has levied on it a certain availability or operational ready requirement. The requirement for chargeable downtime per flight will be established using the procedure presented within this section.

DEFINITIONS - the following definitions apply to this test method:

A = Availability - A measure of the degree (expressed as a probability) to which an aircraft is in the operable and commitable state at the start of a mission, when the mission is called for at an unknown (random) point in time. For this test method, availability is considered synonymous with operational readiness. The aircraft is not considered to be in an operable and commitable state when it is being serviced and is undergoing maintenance.

TOT = Total Active Time in Hours.

Active Time = That time during which an aircraft is assigned to an organisation for the purpose of performing the organisational mission. It is time during which:

1. The aircraft is flying or ready to fly.
2. Maintenance is being performed.
3. Maintenance is delayed for supply or administrative reasons.

DUR = Daily Utilisation Rate - The number of flying hours per day.

AFL = Average Flight Length - Flying hours per flight.

NOF = Number of Flights per day.

DT = Downtime - Time (in hours) during which the aircraft is not ready to commence an assigned mission (i.e. have the flight crew aboard the aircraft).

CMDT = Chargeable Maintenance Downtime - Time (in hours) during which maintenance personnel are working on the aircraft, except when the only work being down would fall under the non-chargeable maintenance downtime (NCMDT) category.

NCMDT - Non-chargeable Maintenance Downtime - Time (in hours) during which the aircraft is not available for immediate flight but the only maintenance being performed is not chargeable. It would include the following:

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[1] Reference - "Introduction to Mathematical Statistics". P Hoel. J Wiley and Sons Inc., 2nd Edition, 1954, pp. 222-229.

1. To correct maintenance or operational errors not attributable to technical orders, contractor furnished training or faulty design.

2. Miscellaneous tasks such as keeping of records or taxiing or towing the aircraft to or from other than the work centre area.

3. Repair of accident or battle damage.

4. Modification tasks.

5. Maintenance caused by test instrumentation.

DDT = Delay Downtime - Downtime (in hours) during which maintenance is required but no maintenance is being performed on the aircraft for supply or administrative reasons. It would include the following:

1. Supply Delay Downtime

    a. Not Operationally Ready Supply (NORS) time.

    b. Item obtainment time from other than the work centre area.

2. Administrative Delay Downtime

    a. Personnel breaks such as coffee or lunch.

    b. No maintenance people available for administrative reasons.

α = The producer's risk: The risk that the producer (or supplier) must take that the hypothesis that a true mean = M_0 will be rejected even though it is true. The desirable value of α must be determined by judgement and agreed upon by the procuring activity and the systems developer. All other things being equal, a smaller value of α will require a larger sample size.

M = The maximum mean chargeable maintenance downtime per flight.

M_0 = The required mean CMDT per flight.

$M-M_0$ = The difference between the maximum mean (M) of the parameter being tested and the specified mean (M_0). This value must be determined in conjunction with a value for β, the consumer's risk. M is a value, greater (or worse) than the specified mean, which the consumer is willing to accept, but only with a small risk or probability (β). If the true mean is in fact equal to the value of M selected, the hypothesis the true mean = M_0 will be accepted, although erroneously, 100 β percent of the time.

β = The consumer's risk. The risk, which the consumer is willing to take, of accepting the hypothesis that the true mean = M_0 when in fact the true mean = M. All other things being equal, a smaller value of β will require a larger sample size.

σ = The true standard deviation of the parameter (CMDT per flight) being tested. *This value, unless it is a specification requirement, will not be known, but an estimate must be made.* (It is assumed that both M and M_0 will have the same value of σ.) The developer's maintainability math model, previous models, or previous data may be used. All other things being equal, a larger value of σ will require a larger sample size.

ASSUMPTIONS - This method requires no assumption as to the probability distribution of chargeable downtime per flight. The method is valid only if the Central Limit Theorem applies, which means that the sample size (number of flights) must be large enough for this theorem to apply. *The sample size must be at least 50, but the actual size is to be determined in accordance with equation B-39.*

DERIVATION OF CMDT PER FLIGHT FROM AVAILABILITY - The requirement for CMDT per flight which will be demonstrated will be determined using the following mathematical derivation:

$$A = 1 - \frac{\text{CMDT} + \text{NCMDT} + \text{DDT}}{\text{TOT}} \qquad \text{(Equation B-30)}$$

$$A(\text{TOT}) = \text{TOT} - \text{CMDT} - \text{NCMDT} - \text{DDT} \qquad \text{(Equation B-31)}$$

$$\text{CMDT} = \text{TOT} - A(\text{TOT}) - \text{NCMDT} - \text{DDT} \qquad \text{(Equation B-32)}$$

$$\frac{\text{CMDT}}{\text{NOF}} = \frac{\text{TOT} - A(\text{T0T}) - \text{NCMDT} - \text{DDT}}{\text{NOF}} \qquad \text{(Equation B-33)}$$

but, $$\text{NOF} = \frac{\text{TOT(DUR)}}{24(\text{AFL})} \qquad \text{(Equation B-34)}$$

therefore,

$$\frac{\text{CMDT}}{\text{NOF}} = \frac{24(\text{AFL})}{\text{DUR}} - \frac{A(24)(\text{AFL})}{\text{DUR}} - \frac{\text{NCMDT}}{\text{NOF}} - \frac{\text{DDT}}{\text{NOF}} \qquad \text{(Equation B-35)}$$

$\frac{\text{CMDT}}{\text{NOF}}$ = CMDT per flight, which will be demonstrated.

Values for DUR an AFL should be those planned for the aircraft during operational use. Values for $\frac{\text{NCMDT}}{\text{NOF}}$ and $\frac{\text{DDT}}{\text{NOF}}$ *are a function of the operational environment. They should be provided to the system developer in the RFP or, if not, must be provided by the developer in his proposal.* The value for availability or operational ready rate should be provided in the RFP.

Example - Following is an example of how a requirement for CMDT per flight $\left(\frac{\text{CMDT}}{\text{NOF}}\right)$ will be derived:

Required A = 0.75

DUR = 2 hours per day

AFL = 4 hours per flight

$$\frac{NCMDT}{NOF} = 0.2 \text{ hours per flight}$$

$$\frac{DDT}{NOF} = 1.0 \text{ hours per flight}$$

Then,

$$\frac{CMDT}{NOF} = \frac{24(4)}{2} - \frac{(0.75)(24)(4)}{2} - 0.2 - 1.0$$

$$\frac{CMDT}{NOF} = 48 - 36 - 0.2 - 1.0$$

$$\frac{CMDT}{NOF} = 10.8 \text{ hours per flight}$$

SAMPLE SIZE - Since the Central Limit Theorem is applied, the expected distribution of the means will take on a normal distribution as in Figure B-7. If the true mean is equal to M_0 and a particular α is desired, the upper distribution (the mean of the distribution will equal M_0) will apply. It is on this basis that an acceptance rule is generated to the effect that if $\overline{X}$ is found to be equal to or less than the value $M_0 + \frac{Z_\alpha \sigma}{\sqrt{n}}$ the item is to be accepted.

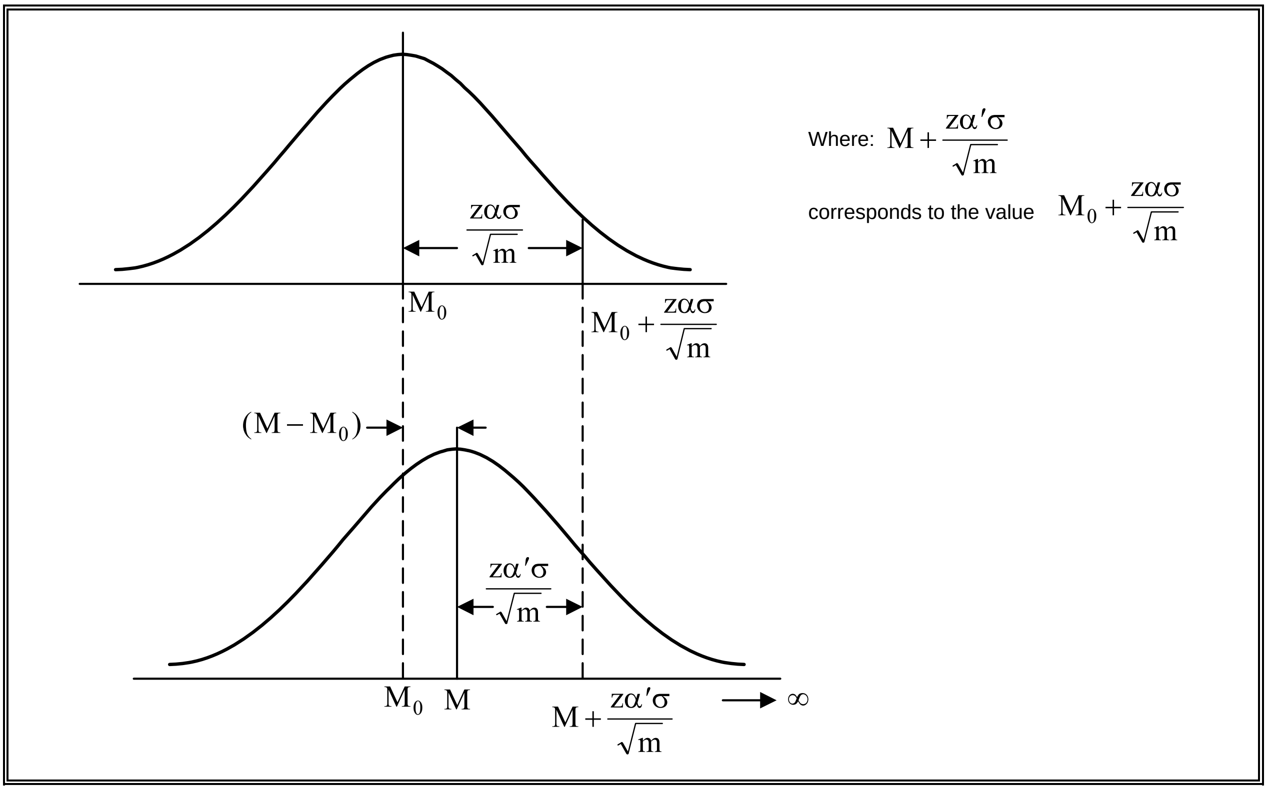

Figure B-7. Distribution of Means

If the true mean is equal to M (which is greater than M_0) the distribution of means will take on a normal distribution with a mean of M as shown in the lower distribution. The value to be used as an acceptance criterion, $M_0 + \frac{Z_\alpha \sigma}{\sqrt{n}}$, corresponds and is equal to a value:

$M + \frac{Z_{\alpha'} \sigma}{\sqrt{n}}$; where α' is a new confidence level

$$M_0 = \frac{Z_\alpha \sigma}{\sqrt{n}} = M + \frac{Z_{\alpha'} \sigma}{\sqrt{n}} ; \qquad \text{(Equation B-36)}$$

where $M = M_0 + (M - M_0)$ (Equation B-37)

$$M_0 + \frac{Z_\alpha \sigma}{\sqrt{n}} = M_0 + M - M_0 + \frac{Z_{\alpha'} \sigma}{\sqrt{n}} \qquad \text{(Equation B-38)}$$

or simplifying, the sample size (n) requirement is:

$$n = \frac{(Z_\alpha - Z_{\alpha'})^2}{\left(\frac{M - M_0}{\sigma}\right)^2} = \frac{Z_\alpha - Z_{(1-\beta)})^2}{\left(\frac{M - M_0}{\sigma}\right)^2} \qquad \text{(Equation B-39)}$$

If this expression should result in n less than 50, then a sample of 50 must be used.

α = Probability of rejection if true mean equals M.

1-α' = β = Probability of acceptance if true mean equals M.

$Z_{\alpha'}$ $Z_{(1-\beta)}$ = Standardised normal deviate as defined.

See table below for relationships between Z_w and α and β, where w = α or 1 - β.

Z_w	.01	.05	.1	.15	.2	.3	.7	.8	.85	.9	.95	.99
	2.33	1.65	1.28	1.04	.84	.52	-.52	-.84	-1.04	-1.28	-1.65	-2.33

$Z_w = Z\alpha$ or $Z_{(1-\beta)}$

Example - Suppose for a requirement of M_0 = 2.0, the following statistical test parameters were agreed to by the procuring activity and the system developer:

$$\alpha = 0.10; Z_\alpha = 1.28; \beta = 0.10; Z_{1-\beta} = 1.28; M - M_0 = 0.30; \sigma = 1.0; \frac{M - M_0}{\sigma} = 0.3$$

Using equation B-39; $n = \frac{(1.28 + 1.28)^2}{(0.3)^2} = \frac{(2.56)^2}{(0.3)^2} = \frac{6.57}{0.09} = 73$

Decision Procedure - The chargeable maintenance downtime (X_i) after each flight will be measured and, at the end of the test, the total chargeable downtime will be divided by the total number of flights to obtain ($\overline{X}$) the sample mean CMDT and the sample standard deviation (s) of CMDT.

$$\overline{X} = \frac{\sum_{i=1}^{NOF} X_i}{NOF} \qquad \text{(Equation B-40)}$$

$$s = \sqrt{\sum_{i=1}^{NOF} \frac{(X_i - \overline{X})^2}{NOF - 1}} = \sqrt{\frac{1}{(NOF - 1}\left[\sum_{i=1}^{NOF} X_i{}^2 - (NOF)\overline{X}^2\right]} \qquad \text{(Equation B-41)}$$

Accept if: $$\overline{X} \le M_0 + \frac{Z_\alpha S}{\sqrt{NOF}} \qquad \text{(Equation B-42)}$$

Reject if: $$\overline{X} > M_0 + \frac{Z_\alpha S}{\sqrt{NOF}} \qquad \text{(Equation B-43)}$$

6.1.1.1.7 **TEST METHOD 6: Test on Manhour Rate[2]** - This test for demonstrating manhour rate (manhours per flight hour) is based on a determination during Phase II test operation of the total accumulative chargeable maintenance manhours and the total accumulative demonstration flight hours. The demonstrated manhour rate is calculated as:

$$\text{Manhour Rate} = \frac{\text{Total Ch arg eable Ma int enance Manhours}}{\text{Total Demonstration Flight Hours}} \qquad \text{(Equation B-44)}$$

If the demonstrated manhour rate is less than or equal to the manhour rate requirement plus a maximum value (ΔMR), by which the demonstrated manhour rate will be permitted to differ from the required manhour rate, then the requirement has been met. ΔMR will be provided, by the procuring activity, as a percentage of the system manhour rate requirement and will be determined based upon such considerations as the expected Phase II duration, and prior experience with similar systems. It is recognised that this demonstration method is non-statistical in nature and does not allow the determination of quantitative producer's and consumer's risk levels. It is for this reason that the ΔMR is provided (in a subjective manner) to minimise the producer's risk.

Normally, all maintenance performed by approved test maintenance personnel during Phase II and documented in appropriate maintenance reports will be the source of data for identifying chargeable maintenance manhours. The procuring activity may elect to terminate the demonstration prior to Phase II completion if sufficient data are collected to project that the requirement will be met.

The manhour rate requirement must pertain to the aircraft configuration provided for in the contract. For Phase II flights conducted with a configuration other than this, an appropriate

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[2] Test Method 6 is intended for use with aeronautical systems and subsystems.

amount of chargeable manhours will be included in calculating the total chargeable manhours. This amount will be based upon the predicted manhour rate associated with the equipment not installed.

Care must be exercised in assuring that the predicted manhour rate pertains to flight time and not equipment operating time. Appropriate ratios of equipment operating time to flight time must therefore be developed.

6.1.1.1.8 **TEST METHOD 7: Test on Manhour Rate - (Using Simulated Faults)[3].** - This test for demonstrating manhour rate (manhours per operating hour) is based on (a) the predicted total failure rate of the equipment used in the formulation of Table B-V (see Section 3.5.2 of Mil-Hdbk-470A, appendix B) and (b) the total accumulative chargeable maintenance manhours and the total accumulative simulated demonstration operating hours. The demonstrated manhour rate is calculated as:

$$\text{Manhour Rate} = \frac{\text{Total Chargeable Maintenance Hours}}{\text{Total Operating Time}} = \frac{\sum_{i=1}^{n} X_{ci} + (PS)}{T} \quad \text{(Equation B-45)}$$

Where:

X_{ci} = Manhours for corrective maintenance task i.

n = Number of corrective maintenance tasks sampled; n must not be less than 30.

MTBF = MTBF of the unit.

(PS) = Estimated average total manhours which would be required for preventive maintenance during a period of operating time equal to n.(MTBF) hours.

$\frac{\sum_{i=1}^{n} X_{ci}}{n} = \overline{X}_c$ = Average number of corrective maintenance manhours per corrective maintenance task.

T = Operating time.

<u>Discussion</u> = When maintenance tasks are simulated, T = n(MTBF), where 1/MTBF = λ_T, the total failure rate of the equipment in question.

$$\frac{\sum_{i=1}^{n} X_{ci} + (PS)}{T} = \frac{\sum_{i=1}^{n} X_{ci} + (PS)}{n \bullet (MTBF)} = \frac{1}{MTBF}\left[\overline{X}_c + \frac{(PS)}{n}\right] \quad \text{(Equation B-46)}$$

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[3] Test Method 7 is intended for use with ground electronic systems where it may be necessary to simulate faults.

All components of (B-46) with the exception of $\overline{X}_c$ can be considered constants. $\overline{X}_c$ can be considered a normally distributed variable when n is large (due to the Central Limit Theorem) with Variance = $\frac{d^2}{n}$.

If $\overline{X}_c$ is normally distributed it can be shown that the function $\frac{1}{MTBF}\left[\overline{X}_c + \frac{PS}{n}\right]$ is also normally distributed around the mean of the manpower rate with Variance = $\left(\frac{1}{n}\right)\left(\frac{d}{MTBF}\right)^2$; assuming d = $\hat{d}$.

Decision Procedure - Therefore, if the manhour rate requirement = μ_R:

Accept if:

$$\overline{X}_c \le \mu_R (MTBF) - \frac{PS}{n} + Z_\alpha \frac{\hat{d}}{n} \qquad \text{(Equation B-47)}$$

where α denotes producer's risk.

6.1.1.1.9 **TEST METHOD 8: Test on a Combined Mean/Percentile Requirement.** - This test provides for the demonstration of maintainability when the specification is couched in terms of a dual requirement for the mean and either the 90th or 95th percentile of maintenance times when the distribution of maintenance time is lognormal.

ASSUMPTIONS - For use as a dual mean and 90th or 95th percentile requirement, the mean must be greater than 10 and less than 100 units of time; the ratio of the 90th percentile maximum value to the value of the mean must be less than two (2); the ratio of the 95th percentile maximum value to the value of the mean must be less than three.

	Maximum Ratio of Percentile to Mean
90th Percentile Value	2
95th Percentile Value	3

Distribution assumptions are as defined above.

DISCUSSION - The test method actually demonstrates the 61st percentile value of maintenance time in combination with either the 90th or 95th percentile values of maintenance time rather than the mean value of maintenance time in combination with either the 90th or 95th percentile values of maintenance time. However, because of the particular characteristic of the lognormal distribution once a 61st percentile value of maintenance time less than X_1 and a 90th or 95th percentile value less than X_2 has been demonstrated, for all practical purposes, a mean value of less than approximately X_1 and a 90th or 95th percentile value less than X_2 have likewise been demonstrated.

A dual requirement on maintainability, assuming a lognormal distribution of repair times, of a maximum value of the Mean in conjunction with either the maximum value of the 90th or 95th percentile of repair time (to be referred to as M_{Max}) results in the definition of various combinations of θs and σs which are acceptable to the dual requirement. (A complete technical description of a lognormal distribution is provided by knowledge of θ and σ, hence all possible lognormal distributions acceptable to the requirements are defined through definition of all possible acceptable values of θ and σ.) See Figure B-8A which defines the acceptable combinations of θ and σ for a Mean of 30 minutes and a 95th percentile (M_{Max}) of 60 minutes.

For the lognormal distribution, it is also possible to structure a dual requirement made up of the maximum values of two percentiles (for example, the 61st percentile of repair time shall be a maximum of 30 minutes and the 95th percentile of repair time shall be a maximum of 60 minutes). This dual requirement also results in the definition of various combinations of acceptable values of θ and σ. See Figure 8-B If a dual percentile requirement could be structured such that the set of acceptable values of θ and σ defined were almost identical to the set of values of θ and σ defined for a given dual Mean and percentile requirement then a demonstration of that dual percentile requirement would in reality also demonstrate the attainment of the dual Mean and M_{Max} requirement. For this particular instance it has been found that under the assumption listed above, almost identical acceptable values of θ and σ are provided for a combined Mean and M_{Max} requirement and a combined 61st percentile (where the value of the 61st percentile is taken equal to the specified value of the Mean) and M_{Max} requirement. See Figure 8-B which defines the values of θ and σ acceptable to a dual 61st percentile (where the value of the 61st percentile is taken equal to a specified mean of 30 minutes) and 95th percentile (where the maximum value of the 95th percentile, M_{Max}, is given as 60 minutes) and Figure 8-C, which is the superimposition of Figure 8-A on Figure 8-B.

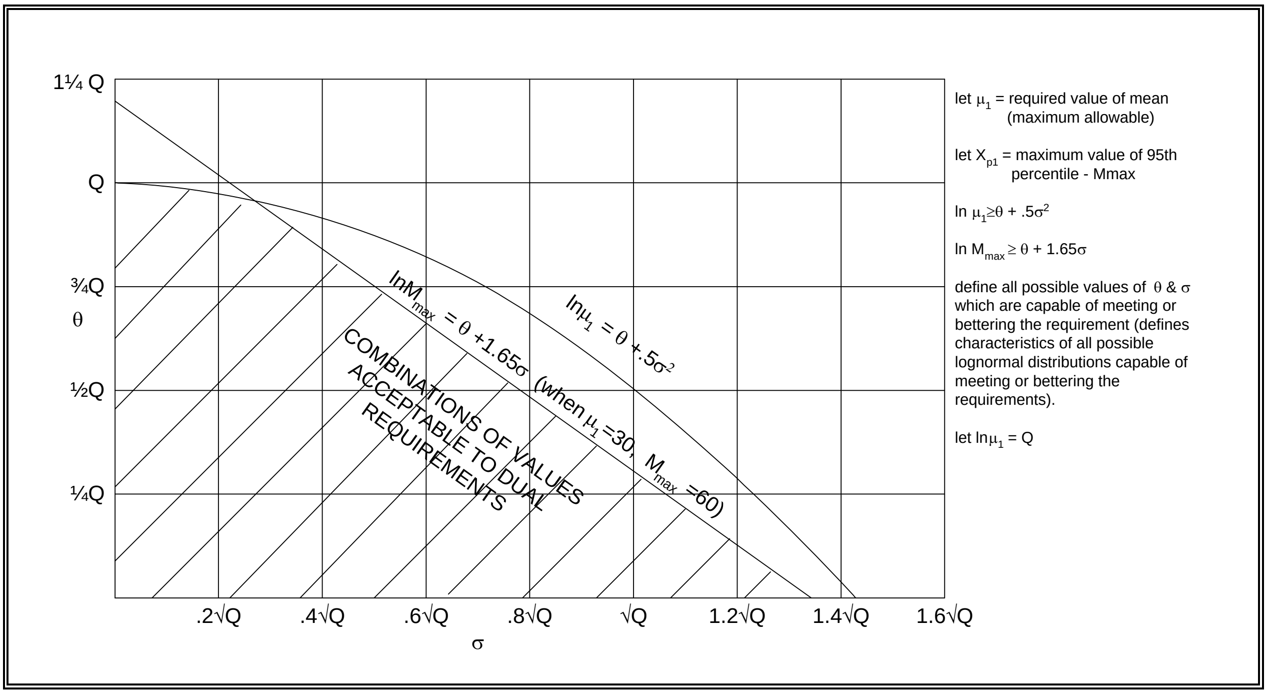

Figure B-8A. Acceptable Combinations of Dual Requirements

Therefore, tests performed to demonstrate the attainment of both the percentiles in question actually demonstrates the attainment of values of θ and σ which are almost identically acceptable to a dual requirement of the Mean and M_{Max}. It follows then that an accept decision relative to both percentiles would also approximately signify an accept decision for a dual Mean and M_{Max} requirement.

Since both percentiles can be considered independent for practical purposes, the same samples can be used for demonstration of both percentiles, therefore, if desired, the tests may be run simultaneously.

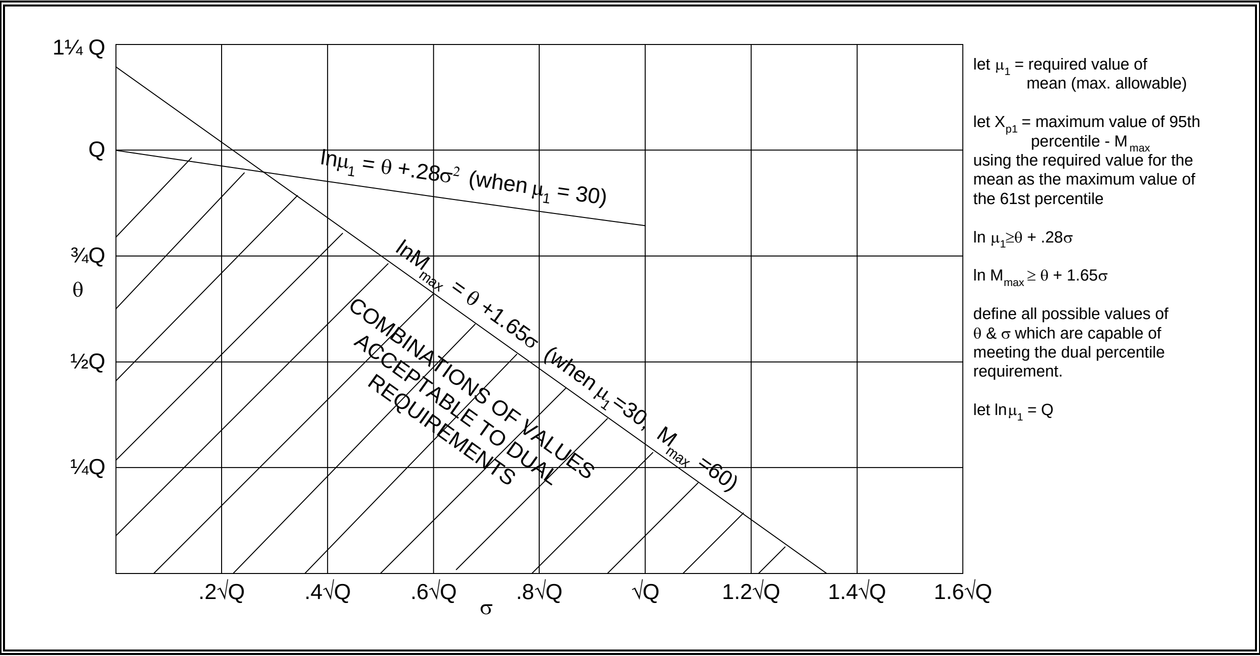

Figure B-8B. Values Acceptable to Dual Requirements of Maximum Values of Two Percentiles

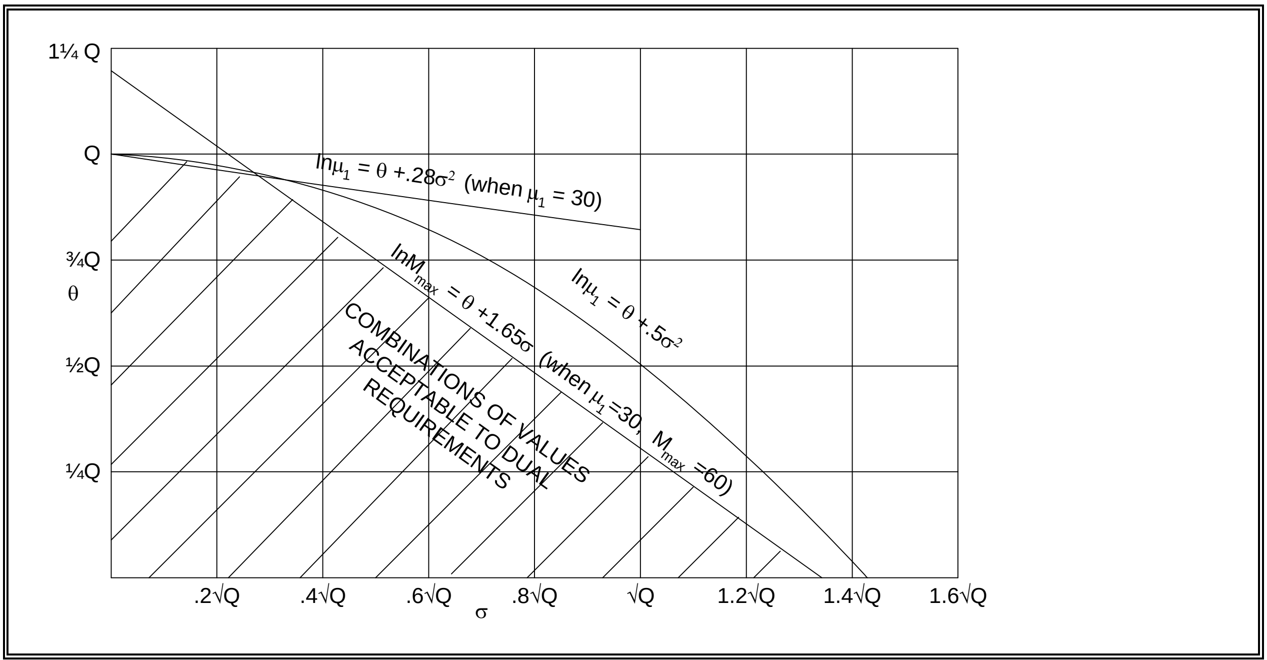

Figure B-8C. Superimposition Figure B-8A and B-8B

PROCEDURE - Sample tasks are to be selected with respect to the procedure defined for variable sample/sequential tests. The same sample tasks may be used simultaneously in the demonstration of both the Mean and M_{Max} requirements. Table B-X[4], Table B-XI[4], and Table BII[4] (which are based upon the sequential probability ratio of proportion) define the

[4] Tables B-X, B-XI and B-XII are appropriate to Test Plans A_1, B_1 and B_2, respectively.

TABLE B-X. PLAN A1 : OBSERVATIONS EXCEEDING THE VALUE OF THE MEAN (OR 61ST PERCENTILE VALUE)

# of Tasks Observed (N)	Accept	Reject	# of Tasks Observed (N)	Accept	Reject
5		5	55	-	-
6		6	56	13	-
7		-	57	-	21
8		-	58	-	-
9		7	59	14	-
10		-	60	-	22
11		-	61	-	-
12	0	-	62	-	-
13	-	8	63	15	23
14	-	-	64	-	-
15	1	-	65	-	-
16	-	9	66	16	-
17	-	-	67	-	24
18	-	-	68	-	-
19	2	-	69	17	-
20	-	10	70	-	25
21	-	-	71	-	-
22	3	-	72	-	-
23	-	11	73	18	-
24	-	-	74	-	26
25	4	-	75	-	-
26	-	12	76	19	-
27	-	-	77	-	27
28	-	-	78	-	-
29	5	-	79	20	-
30	-	13	80	-	28
31	-	-	81	-	-
32	6	-	82	-	-
33	-	14	83	21	-
34	-	-	84	-	29
35	7	-	85	-	-
36	-	15	86	22	-
37	-	-	87	-	30
38	-	-	88	-	-
39	8	-	89	-	-
40	-	16	90	23	31
41	-	-	91	-	-
42	9	-	92	-	-
43	-	17	93	24	-
44	-	-	94	-	32
45	-	-	95	-	-
46	10	-	96	25	-
47	-	18	97	-	33
48	-	-	98	-	-
49	11	-	99	-	-

TABLE B-XI. PLAN (B_1) : OBSERVATIONS EXCEEDING M_{max} - 90 Percentile

# of Tasks Observed (N)	Accept	Reject	# of Tasks Observed (N)	Accept	Reject
2		2	52	-	-
3		-	53	-	5
4		-	54	-	-
5		-	55	-	-
6		-	56	-	-
7		-	57	-	-
8		-	58	-	-
9		-	59	-	-
10		-	60	-	-
11		-	61	-	-
12		-	62	-	-
13		-	63	-	-
14		3	64	-	-
15		-	65	2	-
16		-	66	-	-
17		-	67	-	-
18		-	68	-	-
19		-	69	-	-
20		-	70	-	-
21		-	71	-	-
22		-	72	-	-
23		-	73	-	6
24		-	74	-	-
25		-	75	-	-
26	0	-	76	-	-
27	-	-	77	-	-
28	-	-	78	-	-
29	-	-	79	-	-
30	-	-	80	-	-
31	-	-	81	-	-
32	-	-	82	-	-
33	-	-	83	-	-
34	-	4	84	-	-
35	-	-	85	3	-
36	-	-	86	-	-
37	-	-	87	-	-
38	-	-	88	-	-
39	-	-	89	-	-
40	-	-	90	-	-
41	-	-	91	-	-
42	-	-	92	-	-
43	-	-	93	-	7
44	-	-	94	-	-
45	-	-	95	-	-
46	1	-	96	-	-
47	-	-	97	-	-
48	-	-	98	-	-
49	-	-	99	-	-

TABLE B-XII. PLAN (B_2): OBSERVATIONS EXCEEDING M_{max} - 95 Percentile

# of Tasks Observed (N)	Accept	Reject	# of Tasks Observed (N)	Accept	Reject
2		2	52		3
3		-	53		-
4		-	54		-
5		-	55		-
6		-	56		-
7		-	57	0	-
8		-	58	-	-
9		-	59	-	-
10		-	60	-	-
11		-	61	-	-
12		-	62	-	-
13		-	63	-	-
14		-	64	-	-
15		-	65	-	-
16		-	66	-	-
17		-	67	-	-
18		-	68	-	-
19		-	69	-	-
20		-	70	-	4
21		-	71	-	-
22		-	72	-	-
23		-	73	-	-
24		-	74	-	-
25		-	75	-	-
26		-	76	-	-
27		-	77	-	-
28		3	78	-	-
29		-	79	-	-
30		-	80	-	-
31		-	81	-	-
32		-	82	-	-
33		-	83	-	-
34		4	84	-	-
35		-	85	-	-
36		-	86	-	-
37		-	87	-	-
38		-	88	-	-
39		-	89	-	-
40		-	90	-	-
41		-	91	-	-
42		-	92	-	-
43		-	93	-	-
44		-	94	-	-
45		-	95	-	-
46		-	96	-	-
47		-	97	-	-
48		-	98	-	-
49		-	99	1	-

accept/reject criteria for the values of the required mean, M_{max} (when defined as the maximum 90th percentile value). The number of observations greater than and less than the required values of the Mean and M_{max} must be cumulated separately and compared to the decision values shown in teh tables applicable to the two requirements. When one plan provides an accept decision, attention to that plan is discontinued. The second plan continues until a decision is reached. The equipment is rejected when a decision to reject on either plan has occurred regardless of the status of the other plan. The equipment is accepted only when an accept decision has been reached on both plans. If no accept or reject decision has been made after 100 observations, the following rule applies:

Plan A_1 - Accept only if 29 or less observations are more than the value of the required Mean.

Plan B_1 - Accept only if 5 or less observations are more than M_{Max_c}.

Plan B_2 - Accept only if 2 or less observations are more than M_{Max_c}.

It is recognised and accepted that truncation will somewhat modify probability of acceptance characteristics as described in the following subsection.

The OC Curve - The operating characteristic curve for the test procedure may be determined by mapping the probability of acceptance for various selected points on a diagram of the acceptable and unacceptable regions such as Figure B-8D. (Note that any point can be identified uniquely by the coefficient of Q, where Q = ln (required Mean), on the ordinate and the coefficient of $\sqrt{Q}$ on the abscissa - let the coefficient of Q be denoted as (C) and the coefficient of $\sqrt{Q}$ be denoted as (K) - for example, point B on Figure B-8D can be uniquely located at C = 3/4, K = .4). Each point is also representative of a particular lognormal distribution possessing unique percentiles for the values given for μ_1 (required maximum value for Mean) and M_{Max}, respectively.

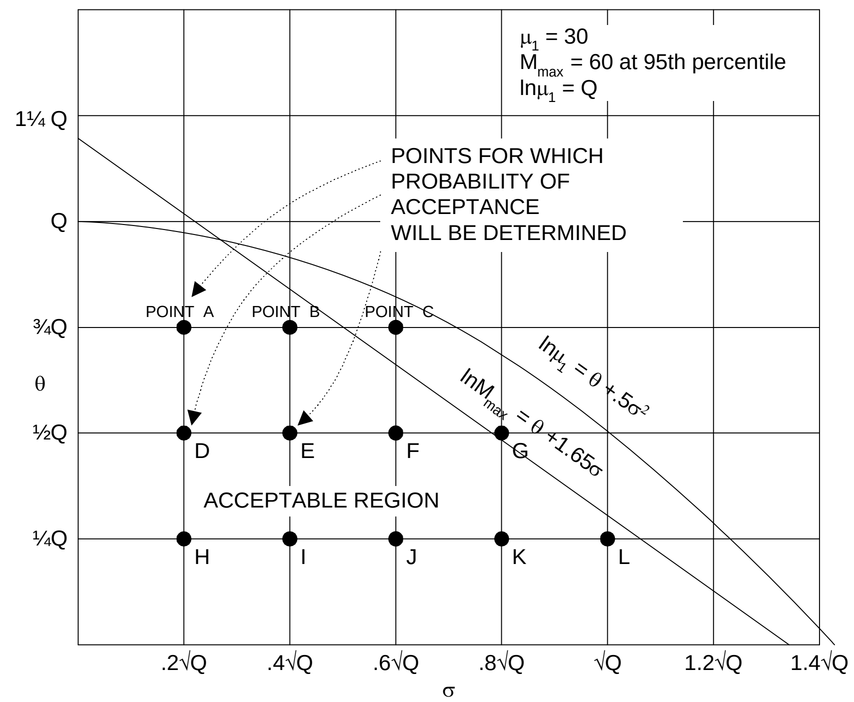

Figure B8-D. OC Curve for Test Method 8

The probability of acceptance relative to any point is equal to the compound probability of passing the percentile test relative to μ_1 (Test A_1) and passing the percentile test relative to M_{Max} (Test B_1 or B_2).

Let P_{A1}, P_{B1} and P_{B2} be the probability of passing test A_1, B_1 and B_2 respectively for any given unique combination of θ and σ (a particular point). P_{A1}, P_{B1} and P_{B2} may be determined by calculating Y_{A1}, Y_{B1} and Y_{B2} from the following equations:

$$Y_{A1} = \frac{\sqrt{Q}(1-C)}{K} \qquad \text{(Equation B-48)}$$

$$Y_{B1} = Y_{B2} = \frac{\ln M_{Max} - CQ}{K\sqrt{Q}} \qquad \text{(Equation B-49)}$$

and entering FIGURE B-8E. Probability of Passing Test A

(for Test A_1) with the calculated value of Y_{A1} and FIGURE B-8F. Probability of Passing Test B (for Test B_1) or FIGURE B-8G. Probability of Passing Test B2

(for Test B_2) with the calculated value of Y_{B1} or Y_{B2}. The corresponding value of probability of acceptance P_{A1} and P_{B1} or P_{B2} (whichever of the B tests are appropriate) is read from each figure and P_{A1} and the appropriate P_{B1} or P_{B2} value are multiplied. The result of this multiplication is the probability of acceptance of a unit having a particular θ and σ characteristic defined by (C) and (K).

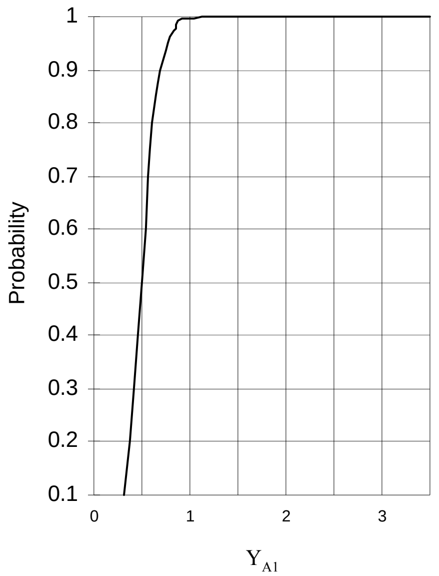

FIGURE B-8E. Probability of Passing Test A

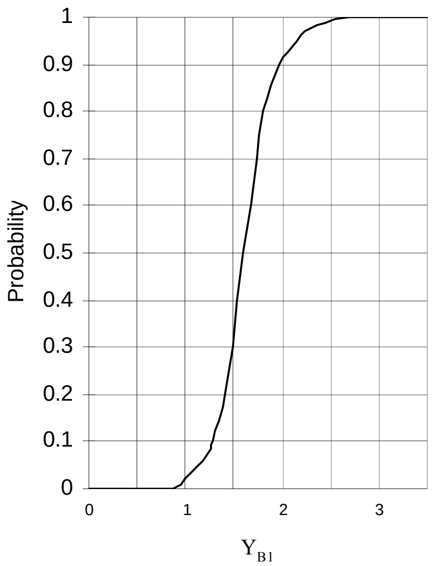

FIGURE B-8F. Probability of Passing Test B_1

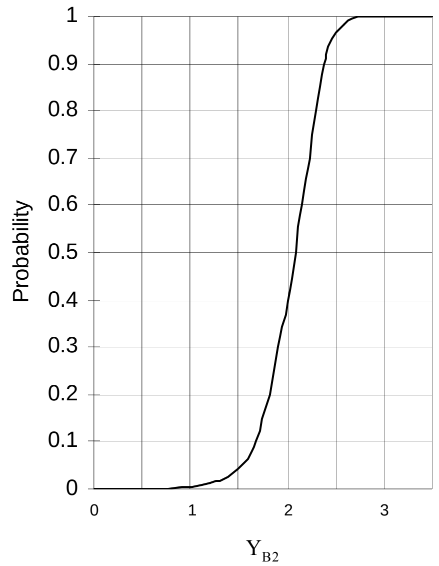

FIGURE B-8G. Probability of Passing Test B_2

Repeating the above for a number of points, as in FIGURE B-9. OC Map Relative to a Given Dual Requirement, defines an operating characteristic map relative to a given dual requirement. Note that probabilities of acceptance always decrease as the point is located upward or to the right and always increase as the point in consideration is located downward or to the left on the figure. Hence, sufficient knowledge of test characteristics can be generated by evaluating relatively few points.

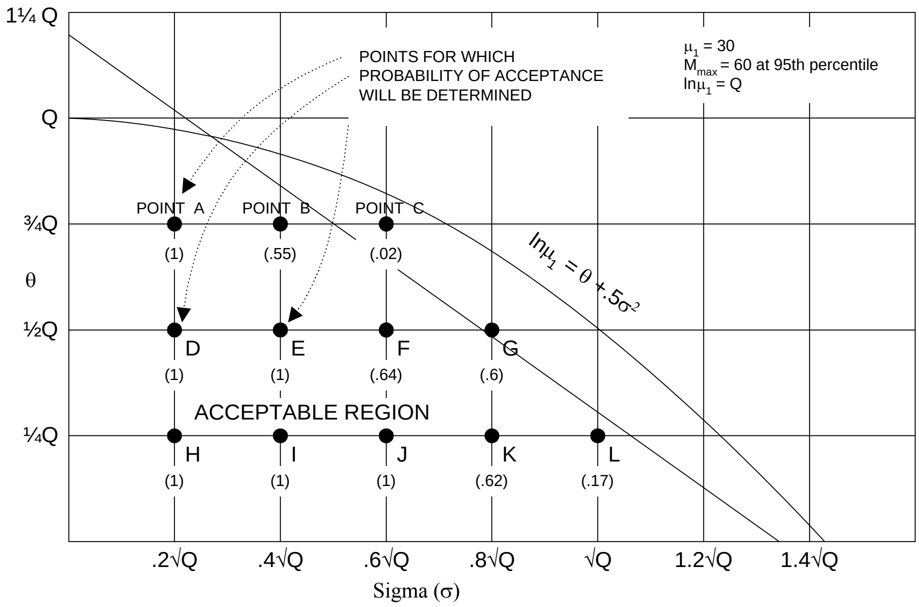

FIGURE B-9. OC Map Relative to a Given Dual Requirement

6.1.1.1.10 **TEST METHOD 9: Test for Mean Maintenance Time (Corrective, Preventive, Combination of Corrective and Preventive) and M_{max}.** This method is applicable to demonstration of the following indices of maintainability: Mean Corrective Maintenance Time (μ_c), Mean Preventive Maintenance Time (μ_{pm}), Mean Maintenance Time (includes preventive and corrective maintenance actions) ($\mu_{p/c}$) and M_{Max} (percentile of repair time).

CONDITIONS OF USE - The procedures of this method for demonstration of μ_c, are based on the Central Limit Theorem. No information relative to the variance (d^2) of maintenance times is required. It may therefore be applied whatever the form of the underlying distribution, provided the sample size is adequate. The maximum sample size is set at 30. The actual sample size (if greater than 30 are required) must be determined for each equipment to be demonstrated, and is usually approved by the procuring activity.

Note: The procedure of this method for demonstrating M_{MaxC} is valid for those cases where the underlying distribution of corrective maintenance task times is lognormal.

QUANTITATIVE REQUIREMENTS - Application of this plan requires identification of the index or indices of interest and specification of quantitative requirements for each. When demonstration involves μ_c or μ_{pm}, or a combination of both, consumer's risks need to be specified. When demonstration involves M_{max_C}, the percentile point which defines the specified value of M_{max_C} is specified. A minimum sample size of 30 corrective maintenance tasks is required for demonstration of corrective maintenance indices. A minimum sample of 30 preventive maintenance tasks is required where demonstration of preventive maintenance indices by sampling is permitted and is to be accomplished by this method.

TASK SELECTION AND PERFORMANCE - Sample tasks are selected in accordance with the stratification procedures outlined in Section 3.5.2. The duration of each is recorded and used to compute the following statistics:

$$\overline{X}_c = \frac{\sum_{i=1}^{n_c} X_{c_i}}{n_c}$$

$$\overline{X}_{pm} = \frac{\sum_{i=1}^{n_{pm}} X_{pm_i}}{n_{pm}}$$

$$D_t = f_c \overline{X}_c + f_{pm} \overline{X}_{pm}$$

$$\overline{X}_{p/c} = \frac{f_c X_c + f_{pm} X_{pm}}{f_c + f_{pm}}$$

$$M'_{max_C} = \text{Antilog}\left[\frac{\sum_{i=1}^{n_C} \ln X_{c_i}}{n_c} + \psi\sqrt{\frac{\sum_{i=1}^{n_C}(\ln X_{c_i})^2 - \frac{\left(\sum_{i=1}^{n_C}\ln X_{c_i}\right)^2}{n_c}}{n_c - 1}}\right]$$

where the Antilog is taken to the Base e and where ψ is the value of the independent variable lognormal function which corresponds to the percentile point at which M_{Max_C} has been established. For the two most common percentile points, 90% and 95%, ψ is 1.282 and 1.645 respectively.

ACCEPT/REJECT CRITERIA - A table of the normal distribution function is consulted for values of ϕ (for a one-tailed test) which corresponds to the specified level of consumer risk β. Table XIII provides values of ϕ which correspond to the most commonly used values of β.

TABLE B-XIII. □ vs. □

ϕ	β
0.84	20%
1.04	15%
1.28	10%
1.65	5%

Accept/reject criteria is computed for each specified index in accordance with the following.

Test for Mean Corrective Maintenance Time (μ_c) - The accept/reject value for μ_c is:

$$\bar{X}_c + \frac{\varphi \hat{d}_c}{\sqrt{n_c}}$$ $\hat{d}_c$ = standard deviation of sample of corrective maintenance tasks.

Accept if μ_c (specified) $\geq \bar{X}_c + \frac{\varphi \hat{d}_c}{\sqrt{n_c}}$

Reject if μ_c (specified) $< \bar{X}_c + \frac{\varphi \hat{d}_c}{\sqrt{n_c}}$

Test for Mean Preventive Maintenance Time (μ_{pm}) - The accept/reject value for μ_{pm} is:

$$\bar{X}_{pm} + \frac{\varphi \hat{d}_{pm}}{\sqrt{n_{pm}}}$$ $\hat{d}_{pm}$ = standard deviation of sample of preventive maintenance tasks.

Accept if μ_{pm} (specified) $\geq \overline{X}_{pm} + \frac{\varphi \hat{d}_{pm}}{\sqrt{n_{pm}}}$

Reject if μ_{pm} (specified) $< \overline{X}_{pm} + \frac{\varphi \hat{d}_{pm}}{\sqrt{n_{pm}}}$

Test for the Mean of all Maintenance Actions ($\mu_{p/c}$) - The accept/reject value of $\mu_{p/c}$ is:

$$\overline{X}_{p/c} + \varphi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 + n_c (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c + f_{pm})^2}}$$

If $\mu_{p/c}$ (specified) $\geq \overline{X}_{p/c} + \varphi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 + n_c (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c + f_{pm})^2}}$. Accept

If $\mu_{p/c}$ (specified) $< \overline{X}_{p/c} + \varphi \sqrt{\frac{n_{pm}(f_c \hat{d}_c)^2 + n_c (f_{pm} \hat{d}_{pm})^2}{n_c n_{pm} (f_c + f_{pm})^2}}$. Reject

Test for M_{Max_c} - The accept/reject value for M_{Max_c} is:

$$M'_{max_c} = \text{Antilog}\left[\frac{\sum_{i=1}^{n_c} (\ln X_{c_i})}{n_c} + \psi \sqrt{\frac{\sum_{i=1}^{n_c} (\ln X_{c_i})^2 - \frac{\left(\sum_{i=1}^{n_c} \ln X_{c_i} \right)^2}{n_c}}{n_c - 1}} \right]$$

where Antilog is to the Base e.

Accept if M_{Max_c} (specified) $\geq M'_{Max_c}$

Reject if M_{Max_c} (specified) $< M'_{Max_c}$.

6.1.1.1.11 **TEST METHOD 10: Tests for Percentiles and Maintenance Time (Corrective Preventive Maintenance).** This method employs a test of proportion to demonstrate achievement of $\tilde{M}_{ct}$, $\tilde{M}_{pm}$, M_{Max_c} and $M_{Max_{pm}}$ when the distribution of corrective and preventive maintenance repair times is unknown.

CONDITIONS OF USE - This method is intended for use in cases where no information is available on the underlying distribution of maintenance task times. The plan holds the confidence level at 75% or 90% as may be desired and requires a minimum sample size (N) of 50 tasks.

QUANTITATIVE REQUIREMENTS - Application of this method requires specification of $\tilde{M}_{ct}$, $\tilde{M}_{pm}$, $M_{Max_{ct}}$ (95th percentile) or $M_{Max_{pt}}$ (95th percentile) and selection of 75% or 90% confidence level.

TASK SELECTION AND PERFORMANCE - Sample tasks are selected in accordance with the stratification procedures. The duration of each task will be compared to the required value(s) of the specified index or indices ($\tilde{M}_{ct}$, $\tilde{M}_{pm}$, $M_{Max_{ct}}$ and $M_{Max_{pm}}$) and recorded as greater than or less than each index.

ACCEPT/REJECT CRITERIA - The item under test shall be accepted when the number of observed task times which exceed the required value of each specified index is less than or equal to that shown in the Table (B-XIV or B-XV) corresponding to each index for the specified confidence level.

Test for the Median - Table B-XIV is a test of the median for corrective and preventive maintenance tasks. The acceptance level is shown for two confidence levels and a sample size (N) of 50 tasks.

TABLE B-XIV[5] Acceptance Table for $\tilde{M}_{ct}$ or $\tilde{M}_{pm}$; Sample Size = 50

Confidence Level	
75%	90%
Acceptance Level	
22	20

Test for M_{max_c} and $M_{Max_{pm}}$ - Table B-XV is a test for M_{Max_c} and $M_{Max_{pm}}$ at the 95th percentile. The acceptance level is shown for two confidence levels and a sample size (N) of 50 tasks.

TABLE B-XV Acceptance Table for M_{Max_c} or $M_{Max_{pm}}$; Sample Size = 50

Confidence Level	
75%	90%
Acceptance Level	
1	0

5 NOTE: Reference for Tables BXIV and B-XV - "Introduction to Statistical Analysis" by Dixon & Massey. Page 230. McGraw-Hill Company. 2nd Edition. 1957.

6.1.1.1.12 **TEST METHOD 11: Test for Preventive Maintenance Times.** This method provides for maintainability demonstration when the specified index involves μ_{pm} and/or $M_{Max_{pm}}$ and when all possible preventive maintenance tasks are to be performed.

CONDITIONS OF USE - All possible tasks are to be performed and no allowance need to made for underlying distribution.

QUANTITATIVE REQUIREMENTS - Application of this plan requires quantitative specification of the index or indices of interest. In addition, the percentile point defining $M_{Max_{pm}}$ must be stipulated when $M_{Max_{pm}}$ is of interest.

TASK SELECTION AND PERFORMANCE - All preventive maintenance tasks will be performed. The total population of PM tasks will be defined by properly weighing each task in accordance with relative frequency of occurrence as follows: Select the particular task for which the equipment operating time to task performance is greatest and establish that time as the reference period. Determine the frequency of occurrence (f_{pm}) of all other tasks during the reference period, where the frequency of occurrence of a given task is a fractional number, the frequency shall be set at the nearest integer. The total population of tasks consists of all tasks with each repeated in accordance with its frequency of occurrence during the reference period.

ACCEPT/REJECT CRITERIA

Test for μ_{pm} - the mean is computed as follows:

$$\mu_{pm}\,(\text{Actual}) = \frac{\sum_{i=1}^{k} f_{pm_i}\,(X_{pm_i})}{\sum_{i=1}^{k} f_{pm_i}}$$

Where: f_{pm_i} is the frequency of occurrence of the i^{th} task in the reference period.

K is the number of different PM tasks.

$\sum f_{pm_i}$ is the total number of PM tasks in the population.

Accept if: μ_{pm} (required) $\geq$ μ_{pm} (actual)

Reject if: μ_{pm} (required) $<$ μ_{pm} (actual)

Test for $M_{Max_{pm}}$ - The PM tasks shall be ranked by magnitude (lowest to highest value). The equipment shall be accepted if the magnitude of the task time at the percentile of interest is equal to or less than the required value of $M_{Max_{pm}}$.

LEAFLET 11/2

EXAMPLE MAINTAINABILITY DEMONSTRATION PLAN

7 INTRODUCTION

7.1 This leaflet provides an example of how a maintainability demonstration plan can be developed for a system.

8 EXAMPLE

8.1 *Problem*

A new communications system has the following maintainability requirements:

The Mean Time to Restore Service (MTRS) is the time taken to diagnose a fault, repair it and restore the system to the level of functionality prior to the fault condition. The MTRS at first line, using a trained maintainer shall not exceed the following values:

a) 20 minutes for system failures.

b) 3 hours for major failures.

c) 5 hours for minor failures.

It is proposed that this value will be verified by a Maintainability Demonstration Test. The communication system has a high level of BITE and electronic equipment and there is little confidence in the maintainability prediction carried out in development. Due to the nature of the equipment there is very little preventive maintenance, and the specification does not include a preventive maintenance requirement.

8.2 *Solution*

8.2.1 *Test Method*

The test method will be based on Test Method 9 of MIL-HDBK-470A. This allows the statistical test to be valid without the need for any assumptions on the distribution of the repair times or their variance. The minimum sample size is set at 30 and the acceptance of the maintainability demonstration test will be deemed to take place when the following conditions are satisfied:

$$\mu_C \geq \overline{X}_C + \frac{\varphi \tilde{d}_C}{\sqrt{n_C}}$$

Where μ_c = *MTRS*

$$\overline{X}_c = \frac{\sum_{i=1}^{n_c} X_{ci}}{n_c}$$

X_{ci} = *Individual task duration*

n_c = *Number of demonstrated task.*

ϕ = *Value corresponding to the specified level of consumers risks (with a consumer risk of 20%,* $\phi = 0.84$*).*

$\tilde{d}_c$ = *Standard deviation of sample of tasks.*

Hence to pass the demonstration it is required that:

$$\mu_c \geq \overline{X}_c + \frac{\varphi \tilde{d}_c}{\sqrt{n_c}} \leq 20 \text{ minutes for system failures.}$$

A consumers risk (β*) of 20% will be applied. The duration of each task will be recorded and used to calculate the following:*

a) *MTRS - Minor Failure.*

b) *MTRS - Major Failure.*

c) *MTRS - System Failure.*

8.2.2 *Accept/Reject Criteria*

To determine the value of Φ *which corresponds to the specified level of confidence, a table of the normal distribution function is consulted. Table 1 provides values of* ϕ *which correspond to the most commonly used values of* β*.*

ϕ	β
0.84	20%
1.04	15%
1.28	10%
1.65	5%

Table 5: Versus

$$\mu_c \geq \overline{X}_c + \frac{\varphi \tilde{d}_c}{\sqrt{n_c}}$$

$\leq$ *3 hours for major failures*

$\leq$ *5 hours for minor failures.*

CHAPTER 11/3

MAINTAINABILITY DEMONSTRATION PLANS

Due to the quantity of graphical content contained within this document it is not possible at present to display Chapter 11 as a web page. Please use the link opposite.