# CHAPTER 8

# RELIABILITY GROWTH MODELS

## CONTENTS

# THE DUANE MODEL

## 1. INTRODUCTION

**1.1** A model that is used frequently in reliability growth assessment is an empirical model first observed by J.T. DUANE (General Electrical Company 1962). It established a relationship between failure rate and testing time for programmes where a continuous effort is made to improve reliability by the prompt introduction of modification following failure.

**1.2** The original relationship was based on failure data for five varied types of systems. These included hydro-mechanical devices, types of aircraft generator and an aircraft jet engine. The model has since been used in a wide variety of applications, including some electronic equipment, and as the postulate for other models (e.g. the AMSAA Model see Serial 8).

**1.3** Much of the literature on reliability growth contains some reference to the Duane model. The aim of this Chapter is to summarise the main features of the model and to consider its application and limitations for reliability growth planning, management and analysis.

**1.4** The key relationship in the Duane Model is that on a log-log plot, the graph of cumulative failure rate verses the cumulative test time is linear. This relationship is known as the "Duane Postulate", in which the negative slope of each line is defined as the growth rate $\alpha$.

## 2. MAIN FEATURES OF MODEL

**2.1** Duane observed an empirical relationship between cumulative failure rate and cumulative operating time given by the expression:

$$\lambda_c = kT^{-\alpha} \quad \text{-----------------------------------------------------------------------------(1)}$$

where $\lambda_c$ = cumulative failure rate at time T (i.e. mean between O and T)

$k$ = constant

$T$ = total operating time

$\alpha$ = a constant, often misleadingly called the growth rate

Taking logarithms in equation (1) yields:

$$\log \lambda_c = (-\alpha)\log T + \log k \quad \text{--------------------------------------------------------------(2)}$$

Thus when $\lambda_c$ is plotted against T on log/log paper, the points will lie on a straight line having slope $(-\alpha)$.

The model is equally applicable to MTBF(M) growth, where $M = 1/\lambda$. Then:

$M_c = KT^{\alpha}$ ------------------------------------------------------------------------------(3)

where $K = 1/k$

and $\log M_c = \alpha \log T + \log K$ ---------------------------------------------------------------(4)

**2.2** These equations are of exactly the same form as those for failure rate except that the power of T is now $= \alpha$. Generally, the MTBF presentation is preferred because the upward slope reflects reliability improvement (i.e. intuitively 'up' is 'good'). In the remainder of this Leaflet, discussion is confined generally to MTBF to simplify the text.

**2.3** Clearly equations (1) to (4) do not hold for T=O because they would give an infinite failure rate or zero MTBF. Also, 'zero' does not appear on a log/log plot. To overcome this feature, some arbitrary datum (or start point) must be selected. Assume that:

$M_s =$ cumulative MTBF at time $T_s$

$T_s =$ operating time (> 1 hour) at which growth plotting is to begin

$M_c =$ cumulative MTBF at time T

$T =$ total operating time

$\alpha =$ the growth slope

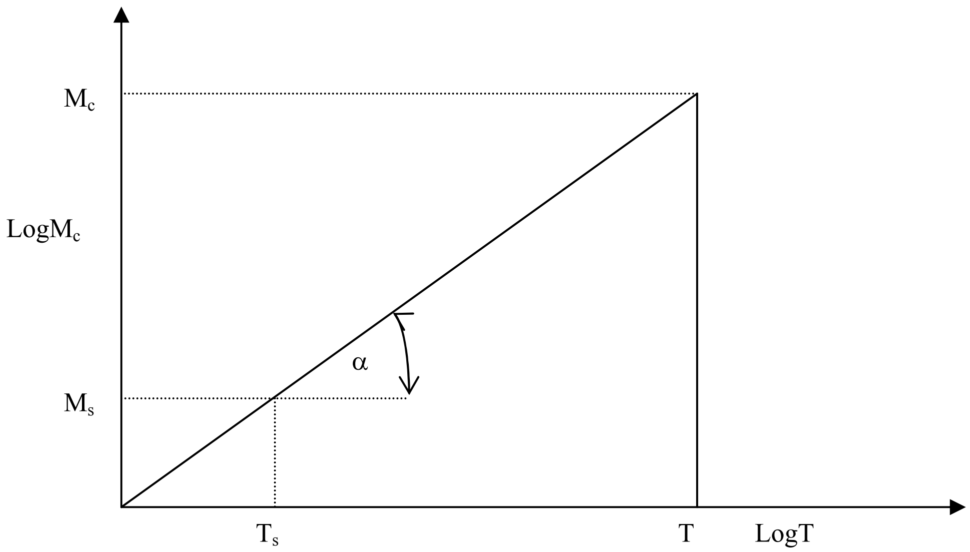



**Figure 1 - Duane Plot Datum**

Referring to Figure 1,

$$\alpha = \frac{\log M_c - \log M_s}{\log T - \log T_s}$$

or $\alpha(\log T - \log T_s) = \log M_c - \log M_s$

or $\log M_c = \alpha(\log T - \log T_s) + \log M_s$ -----------------------------------------------------(5)

Alternatively, equation (5) may be expressed using logarithmic rules as:

$$M_c = M_s\left(\frac{T}{T_s}\right)^{\alpha} = KT^{\alpha}$$ ----------------------------------------------------------------(6)

which is the same as equation (3), but equates to K with $\frac{M_s}{T_s^{\alpha}}$.

which can be rearranged as follows to estimate the test time required in order for the system to meet the target MTBF:

$$T = T_s\left(\frac{Mc}{M_s}\right)^{1/\alpha}$$ ----------------------------------------------------------------(7)

**2.4** Normally, when plotting practical data, the points for low T values will be scattered and not very meaningful. It is usual, therefore, to start the plot after some nominal time (100 test hours is often chosen, i.e.$T_s = 100$) and to use the data during this early period to establish $M_s$. It should be noted, however, that when measuring $M_c$ all failures from T = O must be used and not just those from $T_s$ onward.

**2.5** Assuming growth occurs $(\alpha \rangle O)$, then instantaneous MTBF ($M_i$) at any time T will be greater than the cumulative MTBF ($M_c$) because improvements have been introduced. $M_i$ represents the MTBF which the equipment would have if reliability growth stopped at time T. $M_i$ can be derived by differentiating equation (6), using the definition that $M_c$ is equal to the total time divided by the total number of failures ($M_c = T/F$) as follows:

$$F = \frac{T}{M_c} = \frac{T}{KT^{\alpha}} = \frac{T^{(1-\alpha)}}{K}$$ ----------------------------------------------------------------(8)

and $\frac{dF}{dT} = \frac{(1-\alpha)}{KT^{\alpha}} = \frac{(1-\alpha)}{M_c}$

Since dF/dT is the instantaneous failure rate and the instantaneous MTBF is its reciprocal, then:

$$\frac{dF}{dT} = \frac{1}{M_i} = \frac{(1-\alpha)}{M_c}$$

and $M_i = \frac{M_c}{1-\alpha}$ ---------------------------------------------------------------------------------(9)

Thus the instantaneous MTBF at any point in the test programme is proportional to the cumulative MTBF and can be represented by a line parallel to the cumulative plot and displaced above it by a factor of $\frac{1}{1-\alpha}$ as shown in Figure 2.

$$\log M_i = \alpha(\log T - \log T_s) + \log \frac{M_s}{1-\alpha} \qquad \text{------------------------------------------(10)}$$

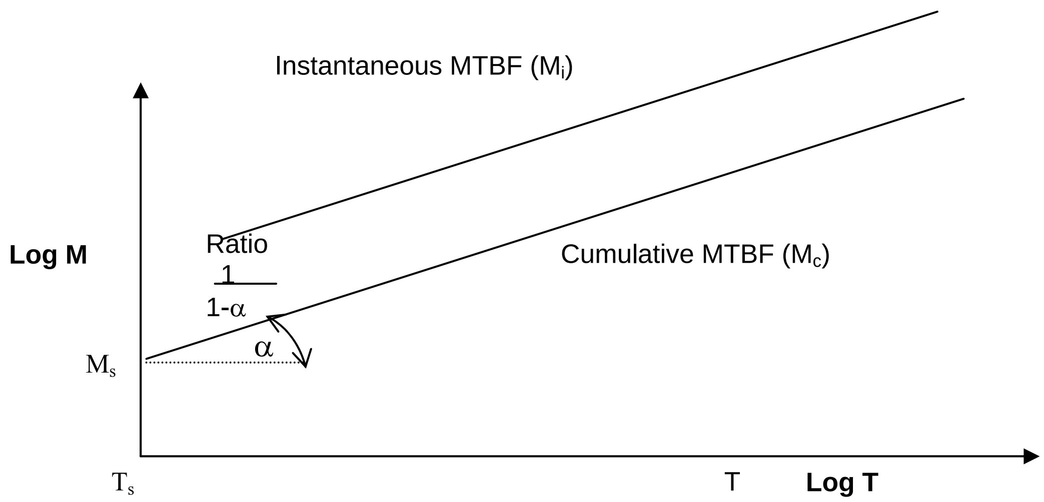



**Figure 2 - Relationship between Cumulative and Instantaneous MTBF**

**2.6** The parameter $\alpha$ is often misleadingly called the 'growth rate'. This would imply that the growth rate is constant with time but this is not so. Figure 3 shows, for the Duane model, a plot of the instantaneous MTBF ($M_i$) against test time using linear scales from which it can be seen that the model gives a high early growth rate followed by a low rate later. This is a feature common to all growth models but one that tends to be obscured by plots on log/log paper.

The true growth rate is the rate at which $M_i$ increases with time, i.e. $dM_i/dT$. This can be obtained from equations (6) and (9) as:

$$\frac{dM_i}{dT} = \left(\frac{\alpha K}{1-\alpha}\right) T^{(\alpha-1)}$$

or $$\frac{dM_i}{dT} = \left(\frac{\alpha K}{1-\alpha}\right) \div T^{(1-\alpha)} \qquad \text{------------------------------------------------------------(11)}$$

**2.7** Typically, $\alpha$ lies between 0.1 and 0.6 and it can be seen, therefore, that the denominator of equation (11) increases indefinitely with T. Hence, the rate of increase of $M_i$ falls with time (as illustrated in Figure 3). Equation (11) may be used as a criterion for determining the time when testing ceases to be 'cost effective'. For example, if it can be established from previous experience that testing ceases to be cost effective when $dM_i / dT \langle \theta$ , then the time at which this condition would be met can be derived from equation (11):

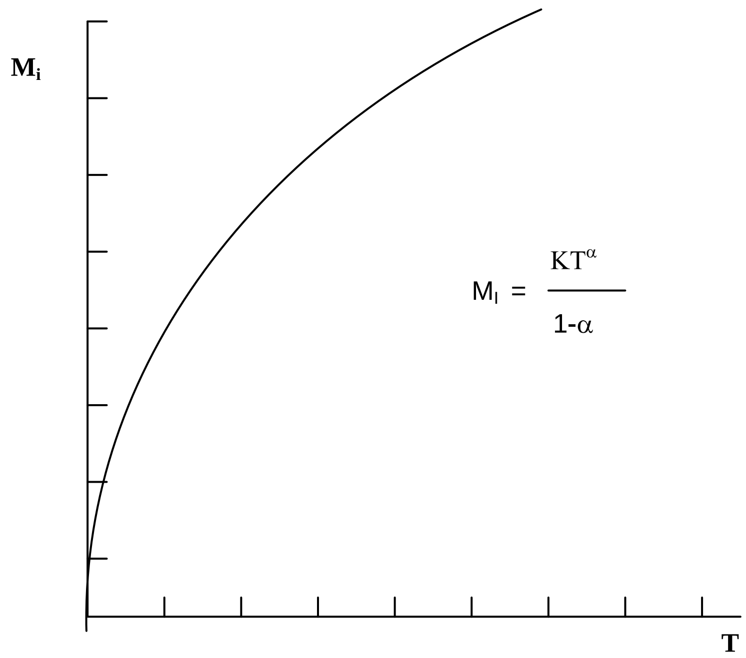



**Figure 3 - Linear Relationship of MTBF with Test Time**

$$\left(\frac{\alpha K}{1-\alpha}\right) \div T^{1-\alpha} = \theta$$

$$\therefore \quad T = \left(\frac{\alpha K}{\theta(1-\alpha)}\right)^{\frac{1}{1-\alpha}} \quad \text{-----------------------------------------------------------------------(12)}$$

This criterion is discussed more fully in Part C Chapter 15, Section 3.6.

**2.8** The practical relationships between the main parameters of the model are illustrated in Figure 4. This shows the considerable effect that both the growth parameter $(\alpha)$ and the ratio of the starting MTBF ($M_S$) to the target MTBF ($M_T$) ($M_S/M_T$) have on the test time necessary to achieve the target. For example, it will be seen from Figure 4 that if $M_s$ = 60 and $\alpha$ = 0.1, then 10000 test hours will only raise the MTBF to just over 100 hours whereas if $\alpha$ = 0.6, the MTBF is raised to 600 hours in only 1000 test hours. Also, if $M_s$ is estimated as 100 hours, then the test time to achieve the predicted MTBF with $\alpha$ = 0.4 is reduced from 9000 hours to about 2400 hours. $M_s/M_T$ and $\alpha$ are important parameters when planning a growth programme.

# 3. PRACTICAL APPLICATION

**3.1** The principal value of any growth model is its ability to forecast the MTBF (or some other reliability characteristic) which can be achieved by a planned growth programme, and later to monitor and assess it. The Duane model is particularly suitable for this through its simplicity of application and its graphical presentation. Above all, it is practical and it has been found that development test data from a wide variety of applications fit the model well. Essentially, it relates failure rate or MTBF, to improvement effort and duration. The duration does not necessarily have to be measured as time and, for example, may be the number of test cycles or number of test firings, etc.

**3.2** The model does have limitations (see Section 1.7) but generally these do not invalidate its use in development testing.

# 4. GROWTH PROGRAMME PLANNING

**4.1** To plan a growth programme for any particular test item, specific values must be adopted for the model parameters. It has already been stressed (paragraph 1.2.6) that test times can vary considerably depending upon the values adopted and different options may need to be examined before arriving at the optimum balance between the variables. However, the prime consideration must be to ensure that the adopted values reflect, as closely as possible, what can be achieved in practice.

**4.2** The following describes how the parameters within the Duane Model are derived:

Target MTBF ($M_T$) is the instantaneous reliability that the system under test is required to meet. The $M_T$ is derived from the system R&M requirements.

**4.3** Growth Parameter $(\alpha)$ is considered to be governed by the intensity and efficiency of the 'test-fix-retest' process (but see also Appendix 1) and generally falls in the range of 0.35 to 0.5. $\alpha = 0.1$ represents a situation in which little consideration is given to reliability during development and any growth is due to the correction of faults discovered during performance tests, on production and in-use. On the other hand, $\alpha = 0.6$ represents a vigorous reliability test programme with speedy corrective action and active management support. It is recommended that, in the absence of more specific information, $\alpha = 0.4$ should be adopted for planning purposes when a project development programme includes reliability testing as specified in this chapter and effective procedures for speedy corrective action are in place. It should be noted that test times are very sensitive to the value of $\alpha$. For example, if $\alpha = 0.4$ then 9000 test hours would raise the MTBF from 60 to 600 hours, if $T_s$ = 100 hours; if $\alpha = 0.41$ only 7600 hours would be required.

**4.4** Starting MTBF ($M_S$) is strictly the cumulative MTBF plotted at Time $T_S$ (see paragraph 1.2.3) but, when planning, it can be regarded as the MTBF which the equipment has at the start of dedicated growth testing (since $T_S$ is small compared with the total growth programme test time). Estimates of likely $M_S$ should be derived from past experience in similar projects, making due allowance for any new design concepts and any other influencing factors. In particular, $M_S$ will depend largely upon the extent and effectiveness of the design evaluation activities before hardware is committed to test (see Part 2 Chapter 6). Beware, however, of adopting over-optimistic values for planning purposes – the adopted values must always reflect the available resources.

**4.5** Test Time. If it were possible for there to be no constraint on test time, the planned values of $\alpha$ and $M_S/M_T$ will determine the planned growth curve and hence the test operating time. This can be assessed graphically or can be computed from equation (10) by substituting $M_T$ for $M_i$ in that equation. Generally, however, there is likely to be some constraint on test time and this will then determine the values of $\alpha$ and/or $M_S/M_T$ which will have to be achieved in practice if the target MTBF is to be met within the limited test time. An example of this is illustrated in Figure 5 (on page 11). The example assumes a $M_T$ of 600 hours and a test time constrained to 6000 hours. From three values of growth parameter $(\alpha)$, the cumulative MTBF's which are equivalent to the target are derived (i.e. $M_c = (1-\alpha)M_T$) (e.g. for a $\alpha$ = 0.4 and an $M_T$ = 600 hours the $M_C$ = 360 hours) and curves drawn downward to determine the $M_S$ which would be required in each case. The number of failures arising (as a function of time) are also assessed to give an indication of the timing and extent of the other resources which will be required, i.e. failure investigation, re-design, etc.

**4.6** Calendar Time. The time (T) in the model is the cumulative operating time. The predicted calendar time of a test programme must allow for the expected down-time involved in waiting, diagnosis, repair, modification, etc. It must be expected that in the early part of the test programme there will be a high proportion of down-time because the failure rate will be high. As the programme progresses, so the proportion of operating time will increase.

## 5. MONITORING THE MTBF USING DUANE

**5.1** Having set the parameters $\alpha$, $M_S/M_T$ and $T_S$ for a particular test programme, the planned cumulative and instantaneous MTBF (or other reliability characteristic) curves are plotted on a log-log scale (see Figure 6 on page 12). When the programme is planned on the basis of discrete test phases, parameter values may need to be set for each test phase. These curves provide the datums against which test results are monitored.

**5.2** Once testing is started, the cumulative MTBF ($M_C$) at the time that a failure occurs is calculated by dividing all relevant failures* which have occurred in the item since T = 0 into the total test operating time T. The values of $M_C$ are then plotted on the log-log scale for comparison with the planned $M_C$ values.

**5.3** It is normal to start plotting $M_C$ when 100 hours of test data have been accumulated (see paragraph 1.2.3) using the earlier data to estimate the starting point, $M_s$. Frequently, the early plots of $M_c$ may be erratic and also show a downward trend. This can be due to a high failure incidence in the early testing period and to delays in modifications coming forward. After a few hundred hours, however, the data points should become more orderly and there should be clear indications of steady growth. This is illustrated in Figure 6.

**5.4** Once sufficient points are available for the purpose, the 'best fit' straight line is drawn through the $M_c$ data points. Most weight must be given to the later points because of the cumulative nature of the plot. The line should be guided principally by these later points, therefore, and fitted to the remaining points, bearing in mind the necessity for weighting.

**5.5** The growth parameter $(\alpha)$ is then measured or calculated, and the $M_i$ line drawn parallel to and displaced by a factor of $1/1-\alpha$ above the $M_C$ line. It is projected until it

* All failures excluding those defined as being not relevant to the MTBF assessment, e.g. secondary, test equipment malfunction, operator induced, etc.

intersects the target MTBF ($M_T$) to show whether the target will be met within the planned timescale ($T_T$) (see Figure 6). If the current and projected achievements represented by the plot differ significantly from those planed, then action must be taken to improve them. For example, the growth parameter is a measure of the corrective action loop. If it is lower than planned, each element of the loop (e.g. data feed-back, re-design, manufacture of modifications, etc) should be investigated.

**5.6** When two or more equipments are tested simultaneously, the test operating times and failures are totalled. However, equipment data must be combined so that they relate to the same starting and operating times and build standard. They will then relate to the same stage of growth even though they may not have started the growth programme at the same calendar date. It is also important to ensure that each equipment is tested for sufficient time to reveal most of the systematic failure modes.

## 6. SETTING THE TARGET MTBF

**6.1** The main purpose of any reliability growth model is to quantify the current reliability achievement at any point during a test programme and to predict the likely achievement by the end of the programme. There will always be some uncertainty about the results achieved because the data may not always fit the model perfectly and the equipment tested may not be typical of the whole population. The principal aim, however, must be to develop the equipment to that stage at which there can be reasonable confidence about the level of reliability that a similar equipment will exhibit.

**6.2** One way of achieving the above aim is to set the target reliability of the equipment under test higher than the level required for demonstration or acceptance purposes, i.e. by having a safety margin. Alternatively, the reliability predictions may be associated with confidence limits. Appropriate methods are described in References 2 and 3.

## 7. LIMITATIONS

**7.1** The Duane model is often criticised theoretically because at zero time it would give zero MTBF or infinite failure rate, which is clearly untrue in practice. The Duane model also implies that given sustained reliability effort, growth will go on indefinitely, albeit at a decreasing rate.

**7.2** These theoretical limitations do not generally invalidate the model for use during development testing since in the very early stages practical results generally exhibit considerable scatter, and the levelling of growth often does not occur, if at all, for several thousands of hours. Always providing the data fits the model reasonable well over the range of major interest during development testing, it can be used.

**7.3** The Duane model is essentially a graphical technique and, as such, may tend to lack accuracy. In particular, the straightness of a series of plotted points and the fitting of a line to cumulative data points can be highly subjective. Also, being plotted on a logarithmic scale, the model is rather insensitive to changes in reliability occurring late in a test programme. Care must be taken not to misinterpret the model and generally some additional monitoring method, such as a Moving Average of Cusums (Part C Chapter 47), should also be used at this time. This will ensure that current trends are more clearly reflected and misinterpretations avoided.

**7.4** The model assumes a uniform level of testing and improvement effort throughout the test programme and the prompt introduction of modification to produce reliability growth. In practice, these conditions may not always be fulfilled. For example, if a number of improvements are held back for embodiment at the same time (i.e. a step change in equipment standard), then the observed growth may fall below target for a while before the instantaneous MTBF shows a sudden improvement. Fluctuations of this sort are another reason for using additional monitoring methods as described in paragraph 1.7.3.

**7.5** When a major re-design of a test item is undertaken during a growth programme, which to some extent creates a new equipment, then a significant discontinuity of slope can be expected in the plot of test results. This will require a new 'datum' to be established for the test plot which takes account of the step-change in MTBF. A method for doing this may be obtained from Reference 4.

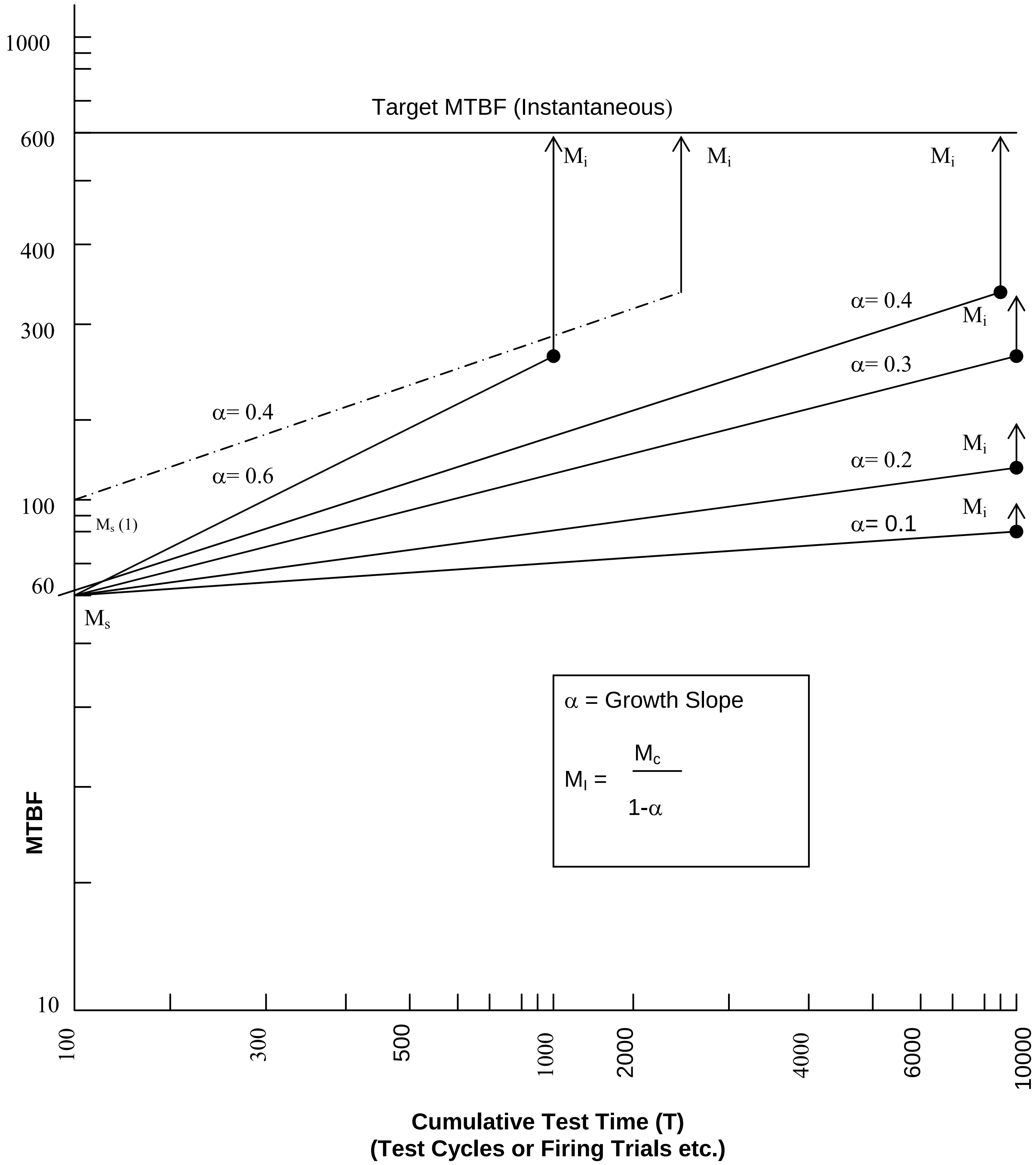
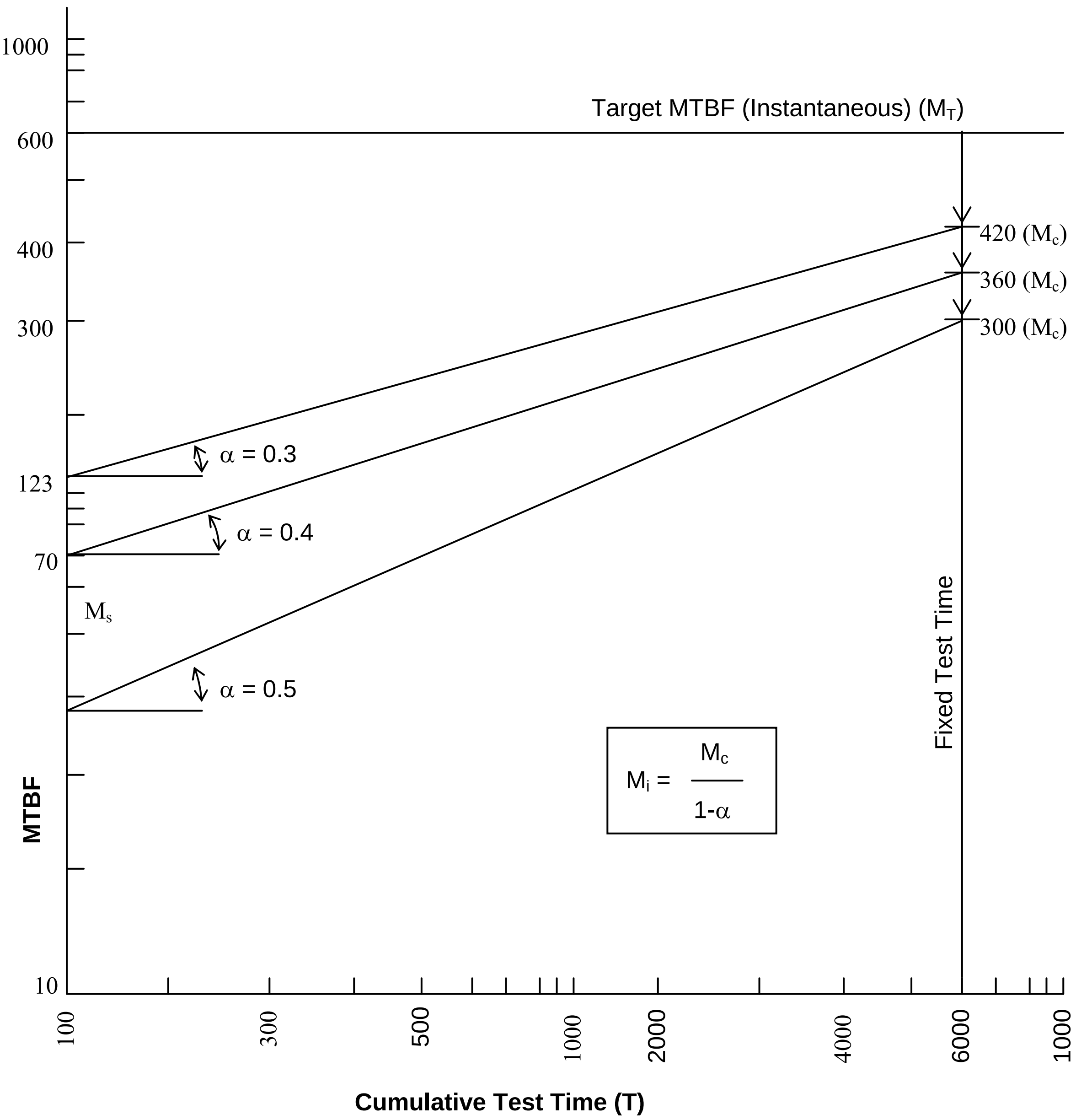
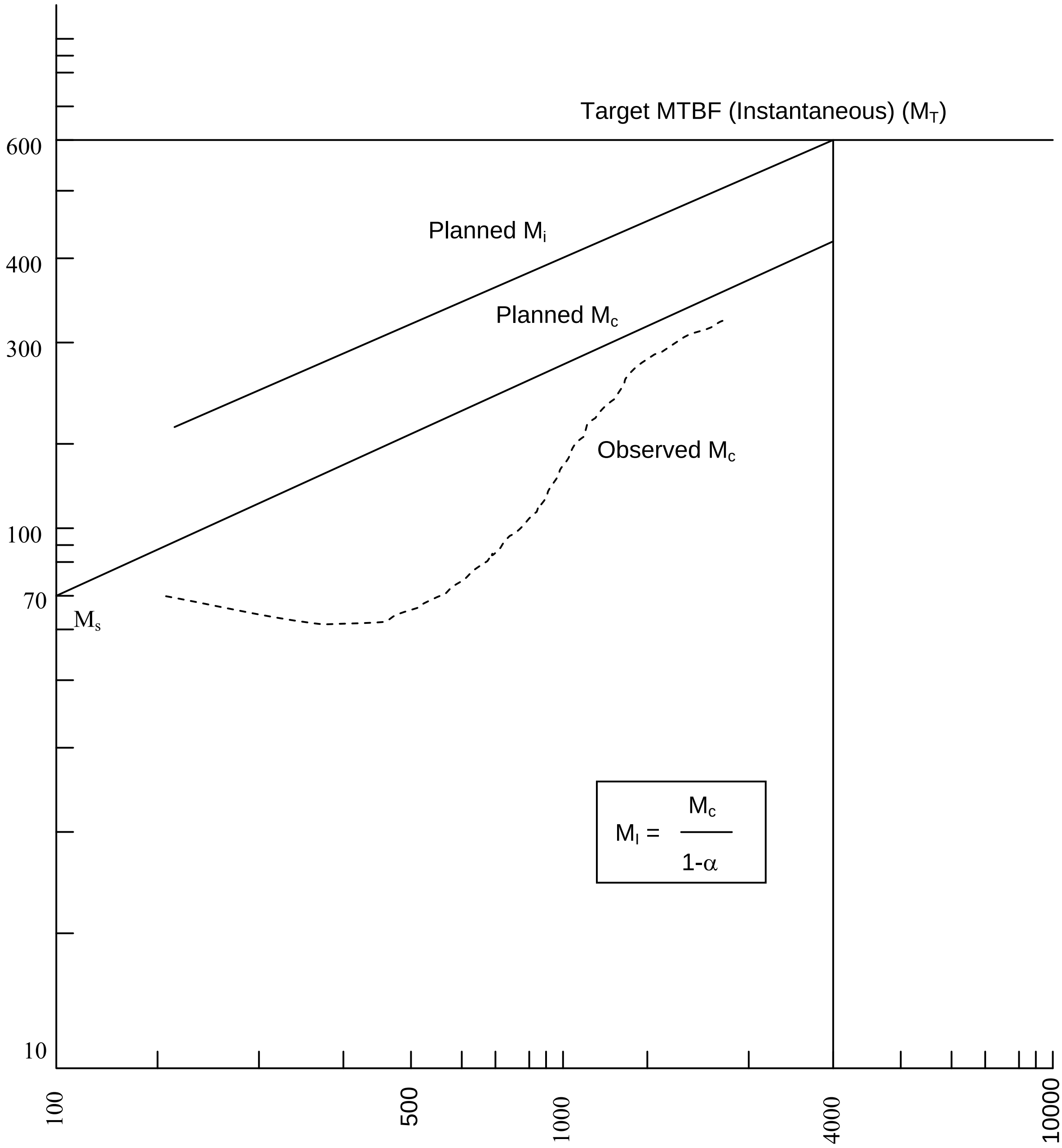



**Figure 4 - Inter-relationship of Parameters in the Duane Growth Model**




| Test Time | Anticipated Number of Failures | | |
|---|---|---|---|
| | $\alpha = 0.3$ | $\alpha = 0.4$ | $\alpha = 0.5$ |
| 0-100 hrs | 1 | 1 | 2 |
| 100-500 hrs | 1 | 2 | 4 |
| 500-1000 hrs | 2 | 2 | 2 |
| 1000-3000 hrs | 4 | 5 | 6 |
| 3000-6000 hrs | 6 | 6 | 6 |
| 0-6000 hrs | 14 | 16 | 20 |

**Figure 5 - Programme and Resource Planning**




Planning Assumptions

$M_T = 500$ hours
$M_s / M_T = 14\%$
Test time constrained to 4000 hours
$\alpha = 0.4$

**Figure 6 - Duane Growth Model – Example of Planned and Observed Plots**

# THE AMSAA RELIABILITY GROWTH MODEL

## 8. INTRODUCTION

**8.1** The AMSAA reliability growth model was developed by Larry H Crow while at the US Army Material Systems Analysis Activity (AMSAA), during the 1970's. The reliability growth pattern for the AMSAA model is exactly the same pattern as for the Duane postulate; however, unlike the Duane postulate the AMSAA model is statistically based.

**8.2** The AMSAA model statistical structure is equivalent to a non-homogeneous Poisson Process (NHPP) model with a Weibull intensity function. This has advantages because the parameters of a NHPP can be estimated on a statistically rigorous basis, confidence intervals can be obtained and goodness of fit test applied.

**8.3** In accordance with Appendix C of Mil-HDBK-189 [Reference 1] the AMSAA model "is designed for tracking the reliability within a test phase and not across test phases" and "assumes that based on the failures and test time within a test phase, the cumulative failure rate is linear on log-log scale. This is a local, within test phase pattern for reliability growth comparable to the global pattern noted by Duane".

**8.4** The AMSAA model analyzes the reliability growth progress within each test phase and can aid in determining the following:

a) Reliability of the configuration currently on test

b) Reliability of the configuration on test at the end of the test phase

c) Expected reliability if the test time for the test phase is extended

d) Growth rate

e) Available confidence intervals

f) Applicable goodness-of-fit tests

## 9. MODEL DEVELOPMENT

**9.1** The transition from Duane to the Weibull Intensity Function form is made by substituting 1-α for β, where β determines the shape and using λ for a scale parameter. λ and β can be determined using maximum likelihood estimators rather than β being assumed to be fixed (as in the Duane Model).

**9.2** Duane first observed that the number of failures accumulated for a system at the total operating time t could be approximated by $\lambda t^{\beta}$, in which $\lambda$ and $\beta$ must be less than one for representation of reliability growth. Crow [Reference 5] formulated a statistical model to describe the pattern of reliability growth. This model provides that the average number of failures accumulated by time t is expressed as $\lambda t^{\beta}$, but the actual number of failures observed to that time is a random variable described by the Weibull process. This development provides a method for calculating statistically valid estimates of the system MTBF if no further improvements are incorporated. This constitutes a means for monitoring reliability growth during the development process.

# 10. BASICS OF THE MODEL

**10.1** This section summarises the basics of the model as described in Mil-Hdbk-189 [Reference 1]. For a more detailed description see Mil-Hdbk-189 [Reference 1].

**10.2** The AMSAA reliability growth model assumes that within a test phase failures are occurring according to a Non-Homogeneous Poisson process, with the intensity of failures during the test phase is represented by a Weibull intensity function $\rho(t)$.

**10.3** The observed cumulative failure rate (C(t)) is defined as

$$C(t) = N(t)/t$$

where the total number of failures (N(t)) accumulated on all test items in the cumulative test time t, is a random variable which follows a Non-Homogeneous Poisson distribution. Where, the probability that exactly n failures occur between the initiation of testing and test time t is

$$P(N(t)=n)=\frac{\theta(t)^n e^{-\theta(t)}}{n!}$$

**10.4** The expected (mean) number of failures by time t ($\theta(t)$) is of the form

$$\theta(t) = \lambda t^{\beta}$$

in which $\lambda$ and $\beta$ are positive parameters.

**10.5** change in time ($\Delta t$), $\rho(t)\ \Delta t$ is approximately the probability of a system failure in the interval ($t, t + \Delta t$). The AMSAA model assumes that

$$\rho(t)=\lambda\beta t^{\beta-1}$$

where t is the cumulative test time, $\lambda$ is the scale parameter, $\beta$ characterises the shape of the graph of the intensity function e.g. when $\beta=1$, $\rho(t)= \lambda$ the homogeneous Poisson or Exponential distribution, when $\beta<1$, $\rho(t)$ is decreasing implying reliability growth and when $\beta>1$, $\rho(t)$ is increasing implying a decrease in system reliability.

**10.6** When production commences the design is fixed and therefore no further reliability improvement is assumed. The constant value of the intensity function of the production model should be approximately equal to the value of the intensity function at the end of development testing. Thus the anticipated MTBF for the production model is equal to the reciprocal of the intensity function at the end of the development phase as follows

$$m(t)=\frac{1}{\lambda\beta t^{\beta-1}}$$

**10.7** Number of failures in an interval from test time 'a' until test time 'b' is a random variable having the Poisson distribution with mean

$$\theta(b) - \theta(a) = \lambda\left(b^{\beta} - a^{\beta}\right)$$

The number of failures occurring in any interval is statistically independent of the number of failures in any interval that does not overlap the first interval and only one failure can occur at any instant.

# 11. GRAPHICAL ASSESSMENT OF RELIABILITY GROWTH

**11.1** This section summarises the graphical assessment methods as described in Mil-Hdbk-189 [Reference 1].

**11.2** Plots derived from the failure data provide a graphic description of test results and can be used to obtain rough estimates of the reliability parameters of interest in the reliability growth process. Two types of graphical methods are used:

a) Average Failure Frequency Plot

b) Cumulative Failure Plot

**11.3** Average Failure Frequency Plot tells the analyst if growth is obviously demonstrated by the data and yields a crude approximation of the intensity function. The Average Failure Frequency Plot is constructed as follows:

a) Divide the elapsed test time into at least three non-overlapping intervals, which can be of unequal length.

b) Calculate the frequency of occurrence of failures within each interval by dividing the number of failures in the interval by its length.

c) Plot the failure frequency as a horizontal line at the appropriate ordinate. The line should extend over the abscissas corresponding to time within the interval. Any significant increasing or decreasing trend in the intensity function should be apparent from this plot.

**11.4** Cumulative Failure Plot is a graph of the observed cumulative number of failures plotted against cumulative test time on full logarithmic paper, which provides crude estimates of the parameters which describe the intensity function. Taking logarithms in the expression for the expected (mean) number of failures by time t yields the result

$$\log \theta(t) = \log \lambda + \beta \log t$$

**11.5** Therefore, the expression for θ(t) is represented by a straight line on full logarithmic paper. A line drawn to fit the data points representing the cumulative number of failures at the time of each failure occurrence is a suitable approximation of the true line. The ordinate of the point of the line corresponding to t=1 is an estimate of the scale parameter $\lambda$. The slope of the line yields an estimate of the shape parameter $\beta$.

**11.6** Appendix C of Mil-Hdbk-189 [Reference 1] contains an example which demonstrates the graphical estimation procedures.

**11.7** The method for estimating the scale and shape parameters and therefore the intensity function within this section is satisfactory for a quick analysis of the data; however, the statistical estimates described in paragraph 2.5 provide a more precise description of the growth process.

## 12. STATISTICAL ASSESSMENT OF RELIABILITY GROWTH

**12.1** The statistical assessment sections summarise the statistical assessment methods as described in Mil-Hdbk-189 [Reference 1].

**12.2** Procedures for point estimation and interval estimation of MTBF are described below for data consisting of failure times from testing terminated at a given time (see Section 2.6) or for data consisting of failure times from testing terminated at the occurrence of a specified number of failures (see Section 2.7). A goodness of fit test to determine whether the model is appropriate to describe the data, is also described below.

**12.3** If the exact times or failure occurrence are unknown, as they are only uncovered during inspection following the testing, it is still possible to utilise the reliability growth model by grouping the data as described in Section 0.

## 13. STATISTICAL ASSESSMENT OF TIME TERMINATED TESTING

**13.1** The procedures described in this section are to be used to analyse data from tests, which are terminated at a predetermined time, or tests, which are in progress with data available through some time. The required data consists of the cumulative test time on all systems at occurrence of each failure as well as the accumulated test time. To calculate the cumulative test time of a failure occurrence it is necessary to sum the test time on every system at that instant. The data then consists of the N successive failure times $X_1$, $X_2$ …., $X_N$ which occur prior to the accumulated test time T.

**13.2** **Point Estimation.** The method of maximum likelihood provides point estimate of the shape parameter $\beta$ and the scale parameter $\lambda$. The shape parameter $\beta$ is estimated as follows:

$$\hat{\beta} = \frac{N}{N \ln T - \sum_{i=1}^{N} \ln X_i}$$

Subsequently, the scale parameter $\lambda$ is estimated as follows:

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}}.$$

And for any time t the intensity function is estimated as follows:

$$\hat{\rho}(T) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}.$$

**13.3** In particular, this holds for T, the accumulated test time. The reciprocal of $\hat{\rho}(T)$ provides an estimate of the MTBF, which could be anticipated if the system configuration remains as it is at time T.

An example of Point Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

**13.4 Interval Estimation.** Interval estimates provide a measure of the uncertainty regarding the demonstration of reliability by testing. For the reliability growth process the parameter of primary interest is the MTBF that the system would exhibit after the initiation of production.

**13.5** The values in Table 1 (page 22) facilitate computation of confidence interval estimates for the MTBF. The table provides two-sided interval estimates on the ratio of the true MTBF to the estimated MTBF for several values of the confidence coefficient. If the number of failures is N and the selected confidence coefficient is $\gamma$, then the appropriate tabular values are $L_{N,\gamma}$ and $U_{N,\gamma}$. The interval estimate of MTBF is

$$\frac{L_{N,\gamma}}{\hat{\rho}(t)} \leq MTBF \leq \frac{U_{N,\gamma}}{\hat{\rho}(t)}$$

**13.6** Because the number of failures has a discrete probability distribution, the interval estimates are conservative and the actual confidence coefficient will be slightly larger than the stated confidence coefficient.

An example of Interval Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

**13.7 Goodness of Fit.** The null hypothesis that a non-homogeneous Poisson process with an intensity function of the form $\lambda\beta t^{\beta-1}$ that properly describes the reliability growth of a particular system is tested by the use of Cramér-von Mises statistic. An unbiased estimate of the shape parameter $\beta$ is used to calculate the goodness of fit as follows for a time terminated test with N failure occurrences:

$$\overline{\beta} = \frac{N-1}{N}\hat{\beta}$$

The goodness of fit statistic is then calculated as follows, with the failure times being ordered so that $O<X_1 \leq X_2 \leq \ldots \leq X_N$:

$$C_N^2 = \frac{1}{12N} + \sum_{i=1}^{N}\left(\left(\frac{X_i}{T}\right)^{\overline{\beta}} - \frac{2i-1}{2N}\right)^2$$

**13.8** The null hypothesis is rejected if the statistic $C_N^2$ exceeds the critical value for the level of significance selected by the analyst. Critical values of $C_N^2$ for the .20, .15, .10, .05 and .01 levels of significance $(\gamma)$ are in Table 2 (page 23). That table is indexed by a parameter labelled M. For time terminated testing M is equal to N, the number of failures. If the test rejects the reliability growth model, an examination of the data may reveal the reason for the lack of fit. Possible causes of rejection include the occurrence of more than one failure at a time or the occurrence of a discontinuity in the intensity function. In the first case, an appropriate procedure may be to group the data as explained in Section 0. In the latter case the data should be treated as described in paragraph 0.

An example of Goodness of Fit Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

# 14. STATISTICAL ASSESSMENT OF FAILURE TERMINATED TESTING

**14.1** The procedures described in this section are applicable to tests that are terminated upon the accumulation of a specified number of failures. The procedures are only slightly different from those used for time terminated testing. The data consists of the N failure times $X_1$, $X_2$ …., $X_N$ expressed in terms of cumulative test time and arranged in ascending order.

**14.2 Point Estimation.** The method of maximum likelihood furnishes point estimates of the shape parameter $\beta$ and the scale parameter $\lambda$. The estimate of $\beta$ is

$$\hat{\beta} = \frac{N}{(N-1)\ln X_N - \sum_{i=1}^{N-1} \ln X_i}$$

Note that this is equivalent to the estimate for time terminated testing with the test time equal to the time of occurrence of the last failure. The scale parameter $\lambda$ is estimated by

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}}$$

as before. The intensity function and mean time between failures are estimated as in paragraph 0. For small sample sizes use of the unbiased estimator $\overline{\beta}$ given in paragraph 0 is advisable.

An example of Point Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

**14.3 Interval Estimation.** An interval estimate of the MTBF that the system would exhibit in the absence of further changes is also available for the case of failure terminated testing. Table 3 (page 24) provides factors for the construction of two-sided interval estimates of the MTBF for several values of the confidence coefficient $\gamma$. As before the interval estimate of MTBF is

$$\frac{L_{N,\gamma}}{\hat{\rho}(t)} \leq MTBF \leq \frac{U_{N,\gamma}}{\hat{\rho}(t)}$$

An example of Interval Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

**14.4 Goodness of Fit**. The hypothesis that the AMSAA model is appropriate can be tested using a Cramér-von Mises statistic. It is important to note the difference in the calculations from those for time-terminated testing. An unbiased estimate of the shape parameter is given by

$$\overline{\beta} = \frac{N-2}{N}\hat{\beta}$$

which is used in the calculation of the goodness of fit statistic. The parameter for indexing that statistic is M which is one less than N, the number of failures. The Cramér-von Mises statistic is then

$$C_M^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left(\left(\frac{X_i}{X_N}\right)^{\overline{\beta}} - \frac{2i-1}{2M}\right)^2$$

**14.5** Table 2 (page 23) provides critical values for use in the test. The model is deemed inappropriate if the statistic $C_M^2$ exceeds the critical value for some specified level of significance $\alpha$ .

An example of Goodness of Fit Estimation is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

## 15. STATISTICAL ASSESSMENT OF GROUPED DATA

**15.1** When the exact time of failure is not known as the failure does not preclude the operation of the equipment, it is possible to predict the MTBF by grouping the failures that have occurred during an interval. It can be assumed that a failure identified during inspection arise in the interval since the last inspection. The total number of failures in the interval between inspections is therefore the sum of the number of failures detected at the time of occurrence and the number of failures found in the inspection. The total number of failures for each interval can then be used to estimate reliability growth in accordance with the AMSAA model providing there are at least three intervals, which do not have to be of equal length.

**15.2** For more information on how to calculate the point estimation of the MTBF and Goodness of Fit for grouped failures see Mil-Hdbk-189 [Reference 1].

## 16. DISCONTINUITIES IN THE INTENSITY FUNCTION

**16.1** This section summarises the assessment method used to calculate the MTBF of a system which has a discontinuous intensity function as described in Mil-Hdbk-189 [Reference 1].

**16.2** The simultaneous introduction of several design changes or some other factor may cause an abrupt change in the intensity function. Such a jump should be detected by a departure from linearity in the full logarithmic plot of cumulative failures, a large change in the level of the average failure frequency, or rejection of the model by a goodness of fit test.

**16.3** The cumulative test time at which a discontinuity has occurred can be determined by inspection from graphs of cumulative failures or average failure frequency. The methods presented above can then be used to estimate the intensity function by use of different parameters for the period before the jump and for the period after the jump. That is, if the discontinuity occurs at time $T_J$, then the intensity function is estimate by

$$\hat{\rho}(t) = \hat{\lambda}_1 \hat{\beta}_1 t^{\hat{\beta}} 1^{-1} \qquad O < t \leq T_J$$

$$= \hat{\lambda}_2 \hat{\beta}_2 (t - T_J)^{\beta_2 - 1} \qquad t > T_J$$

in which $\lambda_1$ and $\beta_1$ are estimated only from failures on or before $T_J$ and $\lambda_2$ and $\beta_2$ are estimated from those failures occurring after $T_J$. Only the second of these equations is needed to estimate the currently achieved value of the intensity function.

An example of calculating the Intensity Function for discontinuities in the Intensity Function is contained in Appendix C of Mil-Hdbk-189 [Reference 1].

## Tables

| N \ $\gamma$ | .80 | | .90 | | .95 | | .98 | |
|---|---|---|---|---|---|---|---|---|
| | L | U | L | U | L | U | L | U |
| 2 | .261 | 18.66 | .200 | 38.66 | .159 | 78.66 | .124 | 198.7 |
| 3 | .333 | 6.326 | .263 | 9.736 | .217 | 14.55 | .174 | 24.10 |
| 4 | .385 | 4.243 | .312 | 5.947 | .262 | 8.093 | .215 | 11.81 |
| 5 | .426 | 3.386 | .352 | 4.517 | .300 | 5.862 | .250 | 8.043 |
| 6 | .459 | 2.915 | .385 | 3.764 | .331 | 4.738 | .280 | 6.254 |
| 7 | .487 | 2.616 | .412 | 3.298 | .358 | 4.061 | .305 | 5.216 |
| 8 | .511 | 2.407 | .436 | 2.981 | .382 | 3.609 | .328 | 4.539 |
| 9 | .531 | 2.254 | .457 | 2.750 | .403 | 3.285 | .349 | 4.064 |
| 10 | .549 | 2.136 | .476 | 2.575 | .421 | 3.042 | .367 | 3.712 |
| 11 | .565 | 2.041 | .492 | 2.436 | .438 | 2.852 | .384 | 3.441 |
| 12 | .579 | 1.965 | .507 | 2.324 | .453 | 2.699 | .399 | 3.226 |
| 13 | .592 | 1.901 | .521 | 2.232 | .467 | 5.574 | .413 | 3.050 |
| 14 | .604 | 1.846 | .533 | 2.153 | .480 | 2.469 | .426 | 2.904 |
| 15 | .614 | 1.800 | .545 | 2.087 | .492 | 2.379 | .438 | 2.781 |
| 16 | .624 | 1.759 | .556 | 2.029 | .503 | 2.302 | .449 | 2.675 |
| 17 | .633 | 1.723 | .565 | 1.978 | .513 | 2.235 | .460 | 2.584 |
| 18 | .642 | 1.692 | .575 | 1.933 | .523 | 2.176 | .470 | 2.503 |
| 19 | .650 | 1.663 | .583 | 1.893 | .532 | 2.123 | .479 | 2.432 |
| 20. | .657 | 1.638 | .591 | 1.858 | .540 | 2.076 | .488 | 2.369 |
| 21 | .664 | 1.615 | .599 | 1.825 | .548 | 2.034 | .496 | 2.313 |
| 22 | .670 | 1.594 | .606 | 1.796 | .556 | 1.996 | .504 | 2.261 |
| 23 | .676 | 1.574 | .613 | 1.769 | .563 | 1.961 | .511 | 2.215 |
| 24 | .682 | 1.557 | .619 | 1.745 | .570 | 1.929 | .518 | 2.173 |
| 25 | .687 | 1.540 | .625 | 1.722 | .576 | 1.900 | .525 | 2.134 |
| 26 | .692 | 1.525 | .631 | 1.701 | .582 | 1.873 | .531 | 2.098 |
| 27 | .697 | 1.511 | .636 | 1.682 | .588 | 1.848 | .537 | 2.068 |
| 28 | .702 | 1.498 | .641 | 1.664 | .594 | 1.825 | .543 | 2.035 |
| 29 | .706 | 1.486 | .646 | 1.647 | .599 | 1.803 | .549 | 2.006 |
| 30 | .711 | 1.475 | .651 | 1.631 | .604 | 1.783 | .554 | 1.980 |
| 35 | .729 | 1.427 | .672 | 1.565 | .627 | 1.699 | .579 | 1.870 |
| 40 | .745 | 1.390 | .690 | 1.515 | .646 | 1.635 | .599 | 1.788 |
| 45 | .758 | 1.361 | .705 | 1.476 | .662 | 1.585 | .617 | 1.723 |
| 50 | .769 | 1.337 | .718 | 1.443 | .676 | 1.544 | .632 | 1.671 |
| 60 | .787 | 1.300 | .739 | 1.393 | .700 | 1.481 | .657 | 1.591 |
| 70 | .801 | 1.272 | .756 | 1.356 | .718 | 1.435 | .678 | 1.533 |
| 80 | .813 | 1.251 | .769 | 1.328 | .734 | 1.399 | .695 | 1.488 |
| 100 | .831 | 1.219 | .791 | 1.286 | .758 | 1.347 | .722 | 1.423 |

For N > 100,

$$L \doteq \left(1 + Z_{\left(.5+\frac{\gamma}{2}\right)} / \sqrt{2N}\right)^{-2} \qquad U \doteq \left(1 - Z_{\left(.5+\frac{\gamma}{2}\right)} / \sqrt{2N}\right) - 2$$

in which $Z_{\left(.5+\frac{\gamma}{2}\right)}$ is the $\left(.5 + \frac{\gamma}{2}\right) - th$ percentile of the standard normal distribution.

**Table 1 - Confidence Intervals for MTBF from Time Terminated Test**

| M \ $\alpha$ | .20 | .15 | .10 | .05 | .01 |
|---|---|---|---|---|---|
| 2 | .138 | .149 | .162 | .175 | .186 |
| 3 | .121 | .135 | .154 | .184 | .23 |
| 4 | .121 | .134 | .155 | .191 | .28 |
| 5 | .121 | .137 | .160 | .199 | .30 |
| 6 | .123 | .139 | .162 | .204 | .31 |
| 7 | .124 | .140 | .165 | .208 | .32 |
| 8 | .124 | .141 | .165 | .210 | .32 |
| 9 | .125 | .142 | .167 | .212 | .32 |
| 10 | .125 | .142 | .167 | .212 | .32 |
| 11 | .126 | .143 | .169 | .214 | .32 |
| 12 | .126 | .144 | .169 | .214 | .32 |
| 13 | .126 | .144 | .169 | .214 | .33 |
| 14 | .126 | .144 | .169 | .214 | .33 |
| 15 | .126 | .144 | .169 | .215 | .33 |
| 16 | .127 | .145 | .171 | .216 | .33 |
| 17 | .127 | .145 | .171 | .217 | .33 |
| 18 | .127 | .146 | .171 | .217 | .33 |
| 19 | .127 | .146 | .171 | .217 | .33 |
| 20 | .128 | .146 | .172 | .217 | .33 |
| 30 | .128 | .146 | .172 | .218 | .33 |
| 60 | .128 | .147 | .173 | .220 | .33 |
| 100 | .129 | .147 | .173 | .220 | .34 |

For M>100 use values for M = 100.

**Table 2 – Critical Value Cramér-Von Mises Goodness of Fit Test**

| N \ $\gamma$ | .80 | | .90 | | .95 | | .98 | |
|---|---|---|---|---|---|---|---|---|
| | L | U | L | U | L | U | L | U |
| 2 | .8065 | 33.76 | .5552 | 72.67 | .4099 | 151.5 | .2944 | 389.9 |
| 3 | .6840 | 8.927 | .5137 | 14.24 | .4054 | 21.96 | .3119 | 37.60 |
| 4 | .6601 | 5.328 | .5174 | 7.651 | .4225 | 10.65 | .3368 | 15.96 |
| 5 | .6568 | 4.000 | .5290 | 5.424 | .4415 | 7.147 | .3603 | 9.995 |
| 6 | .6600 | 3.321 | .5421 | 4.339 | .4595 | 5.521 | .3815 | 7.388 |
| 7 | .6656 | 2.910 | .5548 | 3.702 | .4760 | 4.595 | .4003 | 5.963 |
| 8 | .6720 | 2.634 | .5668 | 3.284 | .4910 | 4.002 | .4173 | 5.074 |
| 9 | .6787 | 2.436 | .5780 | 2.989 | .5046 | 3.589 | .4327 | 4.469 |
| 10 | .6852 | 2.287 | .5883 | 2.770 | .5171 | 3.286 | .4467 | 4.032 |
| 11 | .6915 | 2.170 | .5979 | 2.600 | .5285 | 3.054 | .4595 | 3.702 |
| 12 | .6975 | 2.076 | .6067 | 2.464 | .5391 | 2.870 | .4712 | 3.443 |
| 13 | .7033 | 1.998 | .6150 | 2.353 | .5488 | 2.721 | .4821 | 3.235 |
| 14 | .7087 | 1.933 | .6227 | 2.260 | .5579 | 2.597 | .4923 | 3.064 |
| 15 | .7139 | 1.877 | .6299 | 2.182 | .5664 | 2.493 | .5017 | 2.921 |
| 16 | .7188 | 1.829 | .6367 | 2.144 | .5743 | 2.404 | .5106 | 2.800 |
| 17 | .7234 | 1.788 | .6431 | 2.056 | .5818 | 2.327 | .5189 | 2.695 |
| 18 | .7278 | 1.751 | .6491 | 2.004 | .5888 | 2.259 | .5267 | 2.604 |
| 19 | .7320 | 1.718 | .6547 | 1.959 | .5954 | 2.200 | .5341 | 2.524 |
| 20. | .7360 | 1.688 | .6601 | 1.918 | .6016 | 2.147 | .5411 | 2.453 |
| 21 | .7398 | 1.662 | .6652 | 1.881 | .6076 | 2.099 | .5478 | 2.390 |
| 22 | .7434 | 1.638 | .6701 | 1.848 | .6132 | 2.056 | .5541 | 2.333 |
| 23 | .7469 | 1.616 | .6747 | 1.818 | .6186 | 2.017 | .5601 | 2.281 |
| 24 | .7502 | 1.596 | .6791 | 1.790 | .6237 | 1.982 | .5659 | 2.235 |
| 25 | .7534 | 1.578 | .6833 | 1.765 | .6286 | 1.949 | .5714 | 2.192 |
| 26 | .7565 | 1.561 | .6873 | 1.742 | .6333 | 1.919 | .5766 | 2.153 |
| 27 | .7594 | 1.545 | .6912 | 1.720 | .6378 | 1.892 | .5817 | 2.116 |
| 28 | .7622 | 1.530 | .6949 | 1.700 | .6421 | 1.866 | .5865 | 2.083 |
| 29 | .7649 | 1.516 | .6985 | 1.682 | .6462 | 1.842 | .5912 | 2.052 |
| 30 | .7676 | 1.504 | .7019 | 1.664 | .6502 | 1.820 | .5957 | 2.023 |
| 35 | .7794 | 1.450 | .7173 | 1.592 | .6681 | 1.729 | .6158 | 1.905 |
| 40 | .7894 | 1.410 | .7303 | 1.538 | .6832 | 1.660 | .6328 | 1.816 |
| 45 | .7981 | 1.378 | .7415 | 1.495 | .6962 | 1.606 | .6476 | 1.747 |
| 50 | .8057 | 1.352 | .7513 | 1.460 | .7076 | 1.562 | .6605 | 1.692 |
| 60 | .8184 | 1.312 | .7678 | 1.407 | .7267 | 1.496 | .6823 | 1.607 |
| 70 | .8288 | 1.282 | .7811 | 1.367 | .7423 | 1.447 | .7000 | 1.546 |
| 80 | .8375 | 1.259 | .7922 | 1.337 | .7553 | 1.409 | .7148 | 1.499 |
| 100 | .8514 | 1.225 | .8100 | 1.293 | .7759 | 1.355 | .7384 | 1.431 |

for N>100,

$$L \doteq \left(1+\sqrt{\frac{2}{N}}Z_{\left(.5+\frac{\gamma}{2}\right)}\right)^{-1} \qquad U \doteq \left(1-\sqrt{\frac{2}{N}}Z_{\left(.5+\frac{\gamma}{2}\right)}\right)^{-1}$$

in which $Z_{\left(.5+\frac{\gamma}{2}\right)}$ is the $\left(.5+\frac{\gamma}{2}\right)-th$ percentile of the standard normal distribution.

**Table 3 – Confidence Intervals for MTBF from Failure Terminated Test**

# LEAFLET D8/0

# REFERENCES

1 Proposed MIL-HDBK-XXX (July 1978) Reliability Growth Management (Note: Developed from Ref 2) - US Department of Defense.

2 Proceedings, 1977, US Annual Reliability Symposium (pp 269-274) Duane Growth Model: Estimation of Final MTBF with Confidence limited using a Hand Calculator - P.H. Mead (Ferranti Ltd, Scotland).

3 AMSAA Technical Report No 197 (1977) (AD-A044788) Confidence Interval Procedures for Reliability Growth Analysis - L.H. Crow, US Army Mate.

4 Proceedings of IEEE (US) Annual Symposium on Reliability, 1968 (pp 458-469) Reliability in Real Life - E.O.Codier, IEEE.

5 AMSAA Technical Report No 138 (1974) Reliability Analysis for Comples Repairable Systems - L.H. Crow, US Army Materiel Systems Analysis Activity.

# LEAFLET D8/1

# EFFECT OF DELAYED DESIGN CHANGES ON THE DUANE MODEL

## 1. INTRODUCTION

Most people who have experience of using growth models are aware that the growth parameters are significantly influenced by what happened during the early test period. An example of this is the effect of the time between design changes e.g. design changes tend to be incorporated at specific times (say at build standard changes) rather than continuously in time. It is shown below by means of an example that this can have a significant effect on the growth parameter $\alpha$ . In fact the larger the update periods, the larger the growth is for growth programmes that are similar in all other respects. This means that the common assertion that $\alpha$ is closely related to the growth effort must be treated with some reserve as a programme with delayed design changes can give a spuriously high $\alpha$ value.

## 2. EXAMPLE

**2.1** Assume that the MTBF is constant between design changes which occur at fixed intervals of length $t_i$.

**2.2** Suppose that when $t_i$ = 10 the cumulative MTBF ($M_c$) follows the Duane model with $\alpha$ = 0.4 and passes through the point $M_c(10)$ = 2.5 hours. This line is shown in Figure 1. The theoretical values of instantaneous MTBF ($M_i$) based on this model is shown by the top line in Figure 1. In fact the true $M_i$ values will be a step function, starting with $M_i$ = 2.5 hours for $0<t<10$ hours.

**2.3** Consider now the same growth programme but with updates every 100 hours. For the first 100 hours the MTBF will be 2.5 hours, the starting MTBF ($M_s$). Assuming points are only plotted every 100 hours (it is likely that data will be analysed at such times) the first point will be plotted at t = 100, $M_c$ (100) = 2.5. At 100 hours the design changes will be incorporated and the MTBF will increase to what it was in the 10 hour update case, i.e. about 10.5 hours. At 200 hours, the time of the next update, $M_c(200)$ will be expected to be:

200 $\div$ (total failure to date)

i.e. $$M_c(200) = 200 \div \left( \frac{100}{2.5} + \frac{100}{10.5} \right)$$

i.e. $$M_c(200) = 4.04 \text{ hours}$$

**2.4** Continuing in this way produces the 'dot & dashed' line in Figure 1. The $\alpha$ value for this line is 0.64. Thus changing the design update interval from 10 to 100 hours has changed $\alpha$ from 0.4 to 0.64, a change of 60%. However, the actual growth effort is the same and $M_i(t)$ will be the same in each case for t an integer multiple of 100 hours.

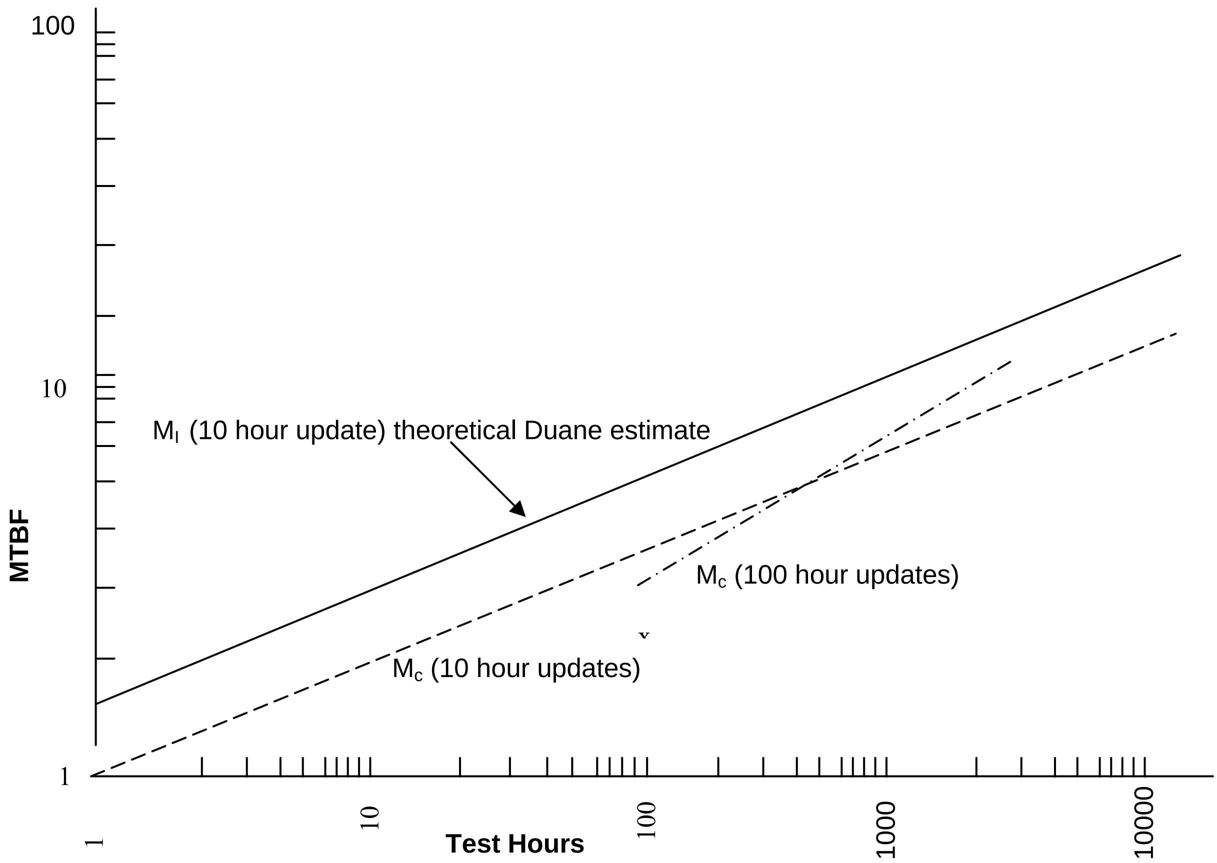



**Figure 1 - Effect of Design Update Interval on the Duane Growth Parameter**

## 3. DISCUSSION

In practice the situation will not be quite this simple as not all design changes are effective and therefore the slower they are incorporated the slower the true growth rate is likely to be. Also the 'dot & dashed' in Figure 1 will not in the long-term follow a straight line. In fact, it will asymptotically approach the line of crosses. However, the example suffices to show the nature of the problem.

# LEAFLET D8/2

# EXAMPLE RELIABILITY GROWTH MODELS

## 1. INTRODUCTION

**1.1** Two Excel based models are provided with this chapter to enable the reader to test the theories and offered in support of reliability growth modelling

**1.2** Each model is write protected and should be downloaded to the host computer, write enabled and saved with the name of choice.

## 2. DUANE

**2.1** The Duane model is used frequently in reliability growth assessment to plot the linear relationship of cumulative failure rate verses the cumulative test time.

**2.2** The model in this workbook is based on an empirical model first observed by J.T. DUANE (General Electrical Company 1962): **Duane Model** (*link*).

## 3. AMSAA

In accordance with Appendix C of Mil-HDBK-189 [Reference 1] the AMSAA model “is designed for tracking the reliability within a test phase and not across test phases” and “assumes that based on the failures and test time within a test phase, the cumulative failure rate is linear on log-log scale. This is a local, within test phase pattern for reliability growth comparable to the global pattern noted by Duane”: **AMSAA Model** (*link*).