

CHAPTER 47

IN-SERVICE DATA COLLECTION

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1 INTRODUCTION

To be completed when the remainder of the chapter is written.

2 CUSUM ANALYSIS

2.1 Introduction

2.1.1 The CUSUM technique offers a straightforward method for the assessment of defect data collected during the operation of a Data Reporting Analysis and Corrective Action System (DRACAS) during development or defect data collected during trialing.

2.1.2 A CUSUM chart monitors the mean value of a variable or process. The main appeal of the CUSUM technique lies in its clear visual presentation of a complex history of trials results. However, it can also be used quantitatively.

2.1.3 CUSUM plots can be used for:

- a) Detecting changes in average levels.
- b) Detecting the time of change.
- c) Obtaining an estimate of current reliability or MTBF.
- d) Predicting future reliability or MTBF over the short term.

2.1.4 The principal application of CUSUMs has been in Process Control and Quality Control; however, the technique is well suited for Reliability Analysis and monitoring. Guided weapon firing results and failures occurring during equipment running are both suitable for CUSUM analysis.

2.1.5 The following text confines itself to the analysis of trials results where either the reliability of one-shot item, or the MTBF or failure rate of operating equipment is of interest.

2.2 Overview

2.2.1 The main feature of a CUSUM analysis is the Cumulative failure rate chart which is produced. The CUSUM method is essentially a graphical technique which produces a graph to show the trend in reliability or failure rate throughout a trial or reporting period. The CUSUM chart is most commonly used to identify:

- a) Changes in Mean Variate Between Failures (MVBF) or Reliability and the point at which they occurred;
- b) Overall MVBF/Reliability;
- c) Current MVBF/Reliability

2.2.2 The CUSUM method may be applied to MVBF or reliability trials of one shot devices such as decoys, guns or missiles. In each case the analyst has a choice of how to generate a CUSUM plot, either using direct failure plotting or usage interval plotting. Direct failure plotting and usage interval plotting are summarised.

2.2.3 Direct Failure Plotting is the most direct method of plotting a CUSUM and consists of examining the data at each failure point and comparing the MVBF indicated by the accumulated usage and failures with the pre-set target MVBF. This method is documented in “Reliability Mathematics and Methods”¹. The main features of this method are that:

- a) This technique is the most detailed as the plot is updated for each failure. The resulting plot can however be cluttered which can reduce the ease with which changes can be identified.
- b) Where data sets contain infrequent events, such as Mission failures, it also has the disadvantage of only plotting when an infrequent failure occurs. If, for example, the failures are all close to the start of the trial, this can leave a long failure free period at end of the trial. Since the CUSUM plot is presented to the last failure only, effectively truncating the trial, a pessimistic answer is indicated since a significant failure free period is ignored.
- c) This is not a problem for frequent events such as Basic failures or failure terminated testing, as the usage to the end of the trial following the last failure will have little or no effect.

2.2.4 Usage Interval Plotting reduces the ‘noise’ on the CUSUM plot and overcomes the problem of plotting infrequent events, data can be plotted at intervals. This method is the most frequently used CUSUM method and is documented in D J Smith’s book “Reliability Maintainability and Risk”². In this method the number of failures are counted for a given usage interval and compared to the number one would expect from the target MDBF. The plot produced in this way is much simpler to interpret and can be plotted beyond the final failure if usage continues. However it can lose detail, particularly at the start of the trial where time to first failure and evidence of infant mortality may be of interest. This matter can be addressed by the careful selection of usage interval.

2.3 Plotting Method

2.3.1 A CUSUM may be plotted in several ways. However, where the mean of interest has a target value associated with it, it is convenient to organise the plot so that when the mean is on target the plot proceeds horizontally (slope = 0), when above the target it slopes up and when below, down. The plot then gives a clear visual indication of whether the current mean is above or below target. As will be clear from the following, it is the *slope* of the plot which measures the mean of interest, *not* the ordinate values of the plot.

2.3.2 Such a CUSUM is constructed by plotting S_n against n , where:

n = number of observations of the variable of interest;

$$S_n = \sum_{i=1}^n (x_i - T) \quad \text{Equation 1}$$

where x_i is the value of the i^{th} observation and
 T is the target for the mean of the x 's.

2.4 Properties of a CUSUM Plot

2.4.1 General

Equation (1) may be written as:

$$\begin{aligned} S_n &= \sum_{i=1}^n x_i - \sum_{i=1}^n T \\ &= \sum_{i=1}^n x_i - nT \end{aligned}$$

now, if we write:

$$\bar{x}_n = \frac{1}{n} \sum_{i=1}^n x_i$$

where \bar{x}_n is the expected or average value of x_i after n trials, then:

$$\bar{S}_n = n\bar{x}_n - nT$$

or $\bar{S}_n = n(\bar{x}_n - T)$ Equation 2

Clearly if $\bar{x}_n = T$, then $S_n = 0$. That is, if the average value of the parameter x is equal to T , then the plot will tend to be horizontal. If \bar{x}_n is greater than T , then its mean slope will be upwards, if it is less, downwards.

For fixed \bar{x}_n , equation (2) is the equation of a straight line with slope $(\bar{x}_n - T)$. The mean slope of a plot therefore gives a measure of the difference between \bar{x}_n and T - the *slope* of the plot is the parameter of interest. It should be noted that although this discussion has assumed fixed \bar{x}_n , when \bar{x}_n changes the slope will change to a value appropriate to the new \bar{x}_n . Local slopes of the plot can therefore be used to monitor changes in \bar{x}_n .

2.4.2 Measuring Slope to Estimate Reliability

It is usual to include on the CUSUM plot a key or legend where sloping lines are drawn to indicate various values of reliability. These can be calculated from equation (2).

To construct a legend, set an 'origin' at some convenient point on the graph away from the main plot. Then, *using the same scale as the main plot*, take some convenient n value (say 10) and compute various \bar{S}_n values for required values of \bar{x}_n using equation (2). Plot these values along the $n=10$ line of the legend axis, and join the points to the legend origin. The lines so drawn are then the slopes appropriate to the various \bar{x}_n values.

2.5 Plotting Trials Results of One-Shot Devices

2.5.1 The plotting technique described here is applicable to any series of trials or events where the outcome is one of two states. Generally these will be success and failure in reliability work (e.g. missile trials).

2.5.2 To produce a CUSUM plot of such reliability data, set:

$$T = \text{target reliability } (0 < T < 1)$$

$x_i = 0$ for item failure, or 1 for item success

2.5.3 This can be seen to satisfy the requirements for the CUSUM plot as follows:

In a series of n trials in which the item reliability is p the expected number of successes is $p.n$; the expected number of failures is $(1-p).n$.

For each success the plot goes up by $(1-T)$, and for each failure it goes down by T .

Hence $\bar{S}_n = p.n.(1-T) - (1-p).n.T$ (where \bar{S}_n is the expected value of S_n)

$$= p.n - p.n.T - n.T + p.n.T$$

$$= p.n - n.T$$

$\therefore \bar{S}_n = n.(p-T)$ Equation 3

2.5.4 Thus if $p = T$, $\bar{S}_n = 0$; i.e. the plot will trend horizontally when the true reliability (p) equals the target reliability (T). Further, for $p > T$ it will trend upwards and for $p < T$ it will trend downwards.

2.6 Monitoring MTBF Using CUSUMS

2.6.1 It is possible to monitor MTBF using a CUSUM chart, and two methods are offered below. (Note that 'time' could in fact be any continuous measure of experience such as distance, etc.).

2.6.2 Method (i)

This method makes use of equation (1) in a straightforward manner. Since the parameter of interest is mean time between failure, then the observations, x , are simply the times between successive failures (excluding repair times). The n of the abscissa indicates the n^{th} such observation.

The advantages of this method is that it is straightforward and 'true' to the basic CUSUM concept of plotting observations. The disadvantages are that accumulated time or experience is not evident from the plot, and the plot is only updated when a failure occurs.

2.6.3 Method (ii)

This method overcomes the disadvantages of method (i) as follows. It should be noted that, in the previous method (Section 2.6.2) that the CUSUM plot would be created by the plot developing upwards at some predetermined slope when trials were successful, and then it would proceed downwards at another predetermined slope if failures occurred.

Thus for MTBF plotting, if time is used instead of n for the abscissa and the plot proceeds upward at some slope (b say) until failure, and drops down a distance h on failure, a plot similar to the 'one-shot' case is produced. This can be used as a CUSUM, as shown below.

Suppose the true MTBF is α say. Then after a time t the expectation is for t/α failures. Thus the expected position of the plot (\bar{S}_t) at time t will be:

$$\bar{S}_t = b.t - \left(\frac{t}{\alpha}\right)h \quad \text{Equation 4}$$

i.e. the amount contributed by the upward slope for time t , less the amount dropped due to failures.

Now the expected mean slope of the plot is $\bar{S}_t/t (= \bar{\rho} \text{ say})$. Therefore from equation (4):

$$\bar{\rho} = b - \frac{h}{\alpha} \quad \text{Equation 5}$$

$\bar{\rho}$ will be zero when $h = b\alpha$. Thus to make the trend horizontal when the MVBF is exactly on target (T)*, make $h = b.T$.

From equation (5):

$$\alpha = \frac{h}{b - \bar{\rho}}$$

Thus, as in other CUSUM plots, the observed mean slope (ρ) can be used to estimate the parameter of interest, in this case the true MTBF (α). As for the 'standard' plot it is useful to construct a legend or key on the chart to indicate the slopes for various MTBF values.

* Note that 'T' represents an MVBF here and not a Reliability Target as in Section 2.5.

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REFERENCES

1. TED QM 079B. Reliability Mathematics and Methods, trials and evaluation. Royal Armament Research and Development Establishment (Chertsey). 21st Jan 87.
2. Reliability, Maintainability and Risk. Practical methods for engineers. Fourth Edition. David J Smith. 1993.

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PLOTTING MVFB BY USAGE INTERVAL (CUSUM)

3 GENERAL

3.1 The method chosen for this example is the plotting of Mean Variate Between Failures (MVBF) by usage interval. The main reasons in favour of applying this particular method are:

- a) the method applies when it is required to plot MVBFs rather than reliability of one shot devices;
- b) the method is consistently based upon usage, and does not terminate at the last failure;
- c) failure terminated testing is rarely used.

3.2 The theory behind the method is discussed in the text of the Chapter. This provides a description of the three most common forms of CUSUM analysis. The data used in this example to build the CUSUM plot is contained in Table 1 and the resulting CUSUM plot is presented in Figure 1.

3.3 The worked example to demonstrate the CUSUM method has been broken down into the following stages:

- a) partitioning of data into usage intervals;
- b) estimation of the Target MTBF;
- c) assessment of the number of failures per interval;
- d) computing deviation and CUSUM for each usage interval;
- e) creating a CUSUM plot;
- f) creating a legend;
- g) interpreting the results.

Each stage is discussed in detail below and can be followed as a step by step guide to creating and interpreting a CUSUM plot.

4 PARTITIONING OF DATA INTO USAGE INTERVALS

4.1 The value of plotting MVBF by usage interval is that the CUSUM plot is updated at each usage interval and therefore can be used to cover a spread of a few failures. It must be noted that the usage interval must reflect the true equipment usage.

Operating Time	Number of Defects	Expected Arisings	Deviation	CUSUM	Operating Time	Number of Defects	Expected Arisings	Deviation	CUSUM
1,000	9	2.99	6.01	6.00	34,000	6	2.99	3.01	-24.80
2,000	6	2.99	3.01	9.00	35,000	0	2.99	-2.99	-27.80
3,000	7	2.99	4.01	13.00	36,000	0	2.99	-2.99	-30.80
4,000	1	2.99	-1.99	11.00	37,000	5	2.99	2.01	-28.80
5,000	1	2.99	-1.99	9.00	38,000	2	2.99	-0.99	-29.80
6,000	0	2.99	-2.99	6.00	39,000	17	2.99	14.01	-15.80
7,000	0	2.99	-2.99	3.00	40,000	15	2.99	12.01	-3.80
8,000	4	2.99	1.01	4.00	41,000	0	2.99	-2.99	-6.80
9,000	0	2.99	-2.99	1.10	42,000	16	2.99	13.01	6.30
10,000	0	2.99	-2.99	-1.90	43,000	37	2.99	34.01	40.30
11,000	2	2.99	-0.99	-2.90	44,000	0	2.99	-2.99	37.30
12,000	0	2.99	-2.99	-5.90	45,000	0	2.99	-2.99	34.30
13,000	0	2.99	-2.99	-8.90	46,000	2	2.99	-0.99	33.30
14,000	0	2.99	-2.99	-11.90	47,000	0	2.99	-2.99	30.30
15,000	0	2.99	-2.99	-14.90	48,000	2	2.99	-0.99	29.30
16,000	0	2.99	-2.99	-17.90	49,000	0	2.99	-2.99	26.30
17,000	0	2.99	-2.99	-20.90	50,000	0	2.99	-2.99	23.30
18,000	2	2.99	-0.99	-21.90	51,000	1	2.99	-1.99	21.30
19,000	5	2.99	2.01	-19.90	52,000	1	2.99	-1.99	19.30
20,000	2	2.99	-0.99	-20.90	53,000	3	2.99	0.01	19.30
21,000	0	2.99	-2.99	-23.90	54,000	3	2.99	0.01	19.30
22,000	4	2.99	1.01	-22.90	55,000	1	2.99	-1.99	17.30
23,000	0	2.99	-2.99	-25.90	56,000	4	2.99	1.01	18.30
24,000	0	2.99	-2.99	-28.90	57,000	0	2.99	-2.99	15.30
25,000	0	2.99	-2.99	-31.90	58,000	1	2.99	-1.99	13.30
26,000	6	2.99	3.01	-28.80	59,000	2	2.99	-0.99	12.40
27,000	4	2.99	1.01	-27.80	60,000	5	2.99	2.01	14.40
28,000	7	2.99	4.01	-23.80	61,000	10	2.99	7.01	21.40
29,000	0	2.99	-2.99	-26.80	62,000	7	2.99	4.01	25.40
30,000	3	2.99	0.01	-26.80	63,000	5	2.99	2.01	27.40
33,000	1	2.99	-1.99	-27.80	64,000	6	2.99	3.01	30.40
34,000	6	2.99	3.01	-24.80	65,000	4	2.99	1.01	31.40
33,000	1	2.99	-1.99	-27.80	66,000	10	2.99	7.01	38.40

Table 1: Input data

4.2 The usage interval should typically be the cumulative usage for the system(s) under test, where usage is expressed as the usual units used to define the equipment reliability and life characteristics. Therefore it may be operating hours as recorded on ETIs (Elapsed Time Indicators) or Central Processing Unit (CPU) Clock for equipment such as radar and communications equipment. It may be expressed as revolutions for

rotating machinery such as propeller shafting, gearboxes etc. It may be calendar time if the equipment usage is measured in time and the systems under test are all used continuously. The usage grouping should be chosen to balance the two opposing requirements to:

- a) have a large number of usage groups (generally not less than 10) so that a progressive and detailed plot is obtained and
- b) maintain a reasonably large usage interval to avoid a large number of null calculations and repetition, although this is not so significant when using a spreadsheet or CUSUM package.

4.3 In the example a trial duration of 66,000 hours has been split into 66 intervals of 1000 hours. This can be seen in Table 1. From the graph it can be seen that the data could easily have been split into 33 intervals of 2000 hours. The chosen grouping is simply more logical and will provide a more detailed plot. The 33 interval plot could have been chosen if the results were required urgently and a rough approximation only was required. Such a plot would have given very similar information to that provided by Figure 1.

5 ESTIMATION OF THE TARGET MTBF

5.1 The target MVBF must be decided before the CUSUM analysis is conducted. This value is necessary as it will provide the scale for the CUSUM graph. As stated in the text of the Chapter, the parameter of interest in a CUSUM graph is the Mean slope of graph over the whole trial or, for example, the last section of the trial. The CUSUM graph will be set up to show a slope of 0° (or a horizontal line) if the trial results demonstrate a MVBF compliant with the Target MVBF. If the trial MVBF and the target MVBF are not similar then the slope of the graph will rise or fall. This is demonstrated by the legend in Figure 2. The Target MVBF should be chosen to reflect:

- a) The MVBF requirement, or
- b) The expected MVBF as a result of predictions or previous experience, or
- c) A combination of both of the above.

5.2 In the example a target MVBF has been defined in terms of operating hours and has been set at 330 hours. This value is assumed to be the required MVBF as defined in a Staff Requirement, Procurement Specification or similar document. It will be seen from the legend (Figure 2) that the slope is 0 when the exhibited MVBF coincides with the Target MVBF. This is achieved by the following steps defined in Sections 4 and 5.

6 ASSESSMENT OF THE EXPECTED AND ACTUAL NUMBER OF FAILURES PER INTERVAL

6.1 Entering the observed number of failures per usage interval is nominally very simple and requires the failure information to be interrogated and grouped by the selected usage intervals. For each usage interval (1000 operating hours in the example) the number of failures are recorded. This results in the information presented in the first two columns of Table 1.

6.2 The expected number of failures per interval is obtained from the expression:

$$\text{Expected Number of Failures} = \frac{\text{Usage interval duration (eg 1000h)}}{\text{Target MVBf (eg 330h)}}$$

Note: The numbers in brackets are provided for clarity and refer to the example.

6.3 The third column of Table 1 contains these values. It will be noted that the expected number of failures per usage interval is a constant value for any given CUSUM analysis.

7 COMPUTING DEVIATION AND CUSUM FOR EACH USAGE INTERVAL

7.1 The deviation is shown in column 4 of Table 1 and is the difference between the expected and observed number of failures for each usage interval. A running total of these deviations is maintained in column 5 of Table 1 and is termed the CUSUM or CUmulative SUM of the differences between the expected and observed failures.

8 CREATING A CUSUM PLOT

8.1 The CUSUM value is plotted graphically on the Y axis against the cumulative usage on the X axis as shown in Figure 1. The graphical method is simple and uses linear-linear graph paper.

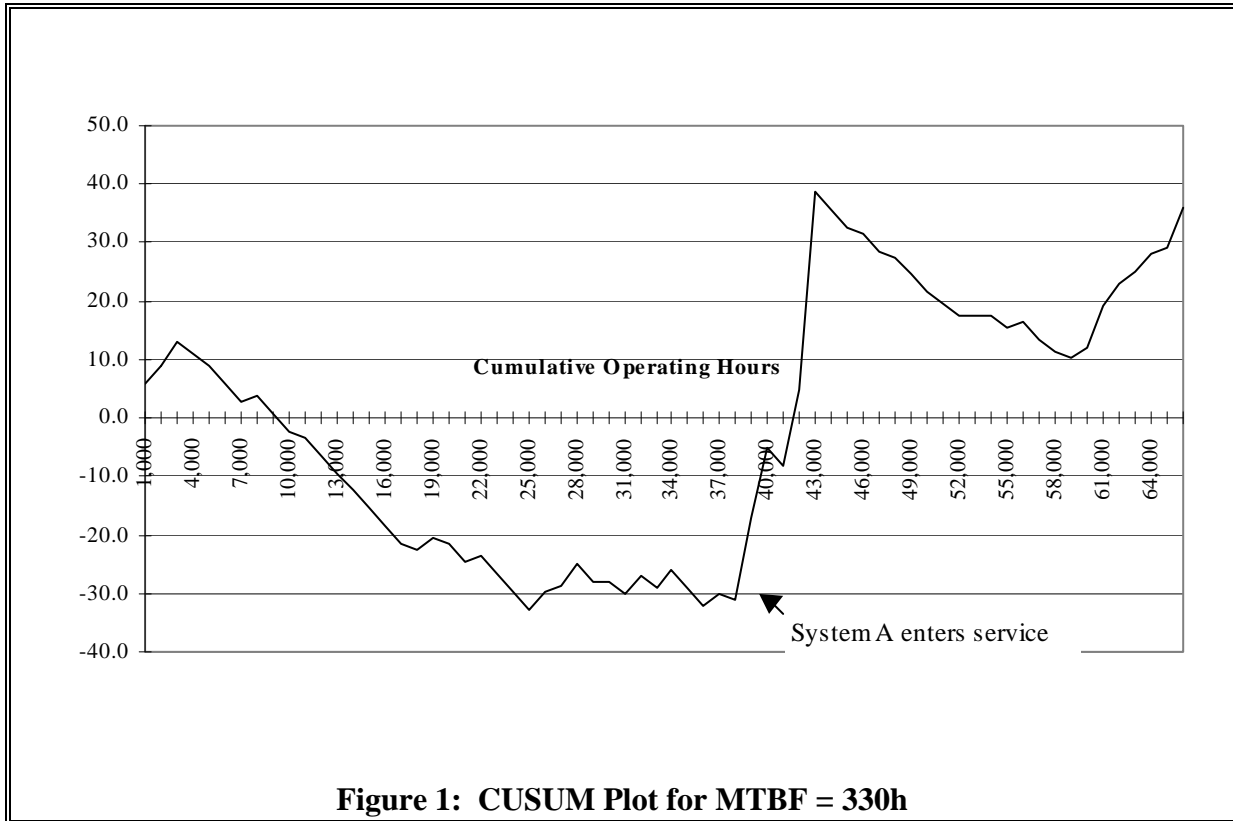


Figure 1: CUSUM Plot for MTBF = 330h

9 CREATING A LEGEND

9.1 The legend is created as described in the text of the Chapter. In summary, this process is conducted manually to produce a number of lines whose slopes are representative of parts of the graph which are considered to be of interest. The following section describes how to create part of the legend from the example which shows an MTBF of 60 hours.

9.2 The first step is to decide the range of data points which represent the slope to be shown on the legend. In the case of this example it is the part of the graph between 38,001 hours and 44,000 hours.

9.3 Establish the MVBF during the period in question by dividing the usage period by the number of observed failures. In the example the usage period is 43,000-38,000 hours = 5,000 hours approx. From Table 1 it will be seen that the numbers of failures recorded in Table 2 occurred:

Usage period	Failures
38,001 - 39,000	17
39,001 - 40,000	15
40,001 - 41,000	0

41,001 - 42,000	16
42,001 - 43,000	37
Total:	85

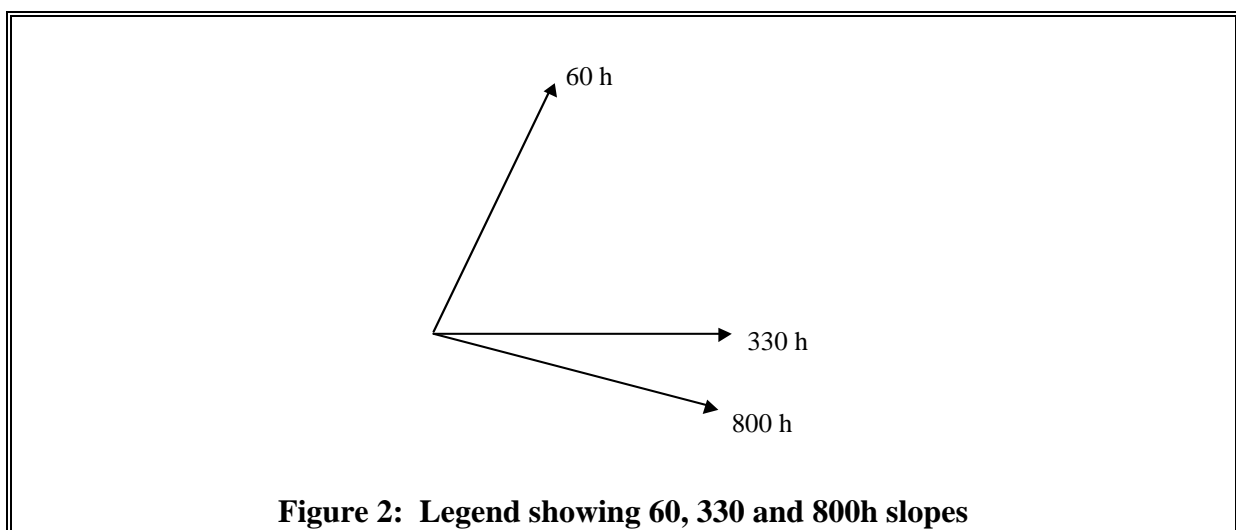
Table 2: Failures in set periods

9.4 This gives 85 failures over 5,000 hours which gives an MTBF of 59 hours. Due to the inherent inaccuracies of this method and the opportunity to consider different points on the graph it is reasonable to consider this slope as approximately 60 hours MTBF, and this is shown on the legend to the graph at Figure 2.

9.5 The legend is created by:

- a) drawing a line between the ordinates for the ends of the period under consideration;
- b) moving that line to the legend (maintaining its gradient); and
- c) scaling the length as required.

9.6 Creation of the legend after the CUSUM graph has been created is beneficial, since the first run of a CUSUM plot may show the expected and target MVBFs to differ by a large degree. The result would be a graph plot which is continually above or below the X axis. It is advisable to redraw such a graph with the expected MVBF revised to that indicated by the original graph. The general design criteria for CUSUM graphs is to minimise the mean deviation (above and below) of the line about the X axis. This gives the most accurate form of plot. The reason for this is that the further the plot varies from the X axis the lower the resolution and hence accuracy of the plot.



10 INTERPRETING THE RESULTS

10.1 The power of the CUSUM analysis comes from its primary strength which is simplicity. The plot represents real cumulative usage, undisguised by Log plotting or complex conversion of the data. The changes of MVBF are identified clearly as changes in the gradient of the graph. These may be traced back directly to a trial or programme plan to identify any changes which may have taken place to explain the change(s) in MVBF.

10.2 Changes in gradient will commonly be due to factors such as:

- a) Components suffering premature wear out will tend to lead to a reduction in the MVBF. This has been observed during trialing by MOD Land Systems.
- b) Changes to the equipment specification, such as implementation of modifications as a result of failures during the previous section of the trial. If the design modifications are well thought out then the modifications will lead to an increase in the MVBF.
- c) Changes to the programme status. For example the period when an equipment is commissioned or enters service will typically see a reduction in the MVBF until new problems are resolved. This is shown in Figure 1 with the system entering service at 40,000 hours.