# CHAPTER 38

# MARKOV MODELLING

## CONTENTS

# 1 INTRODUCTION

## 1.1 General

**1.1.1** Part C Chapter 30 of this Manual describes Reliability Block Diagram analysis, which is perhaps the most familiar method of analysing the functional relationship of a system from a reliability standpoint. Part C Chapter 35 describes analytical means of performing calculations using RBD's, but these calculations are only valid under certain restrictive assumptions (e.g. independence of 'blocks', no queuing for repair, etc.).

**1.1.2** There are modelling techniques that may be used as tools to overcome some of these restrictions. They may be grouped into two general types, the Markov model and the simulation or Monte Carlo model (Part D Chapter 4). Both models use the RBD as a means of representing the functional elements of a system and their interrelationships. Both are also stochastic modelling techniques, meaning that they can deal with events having an element of chance and hence a set of possible outcomes as opposed to deterministic modelling, where a single outcome is derived from a defined set of circumstances. In R&M engineering stochastic modelling is used to describe a system's operation with respect to time. The sub-system failure and repair times typically become the random variables.

**1.1.3** For many years the size and cost of computers capable of running all but the simplest simulation models limited their use despite the advantages set out in Table 1. In recent years, however, increases in computing power and associated cost decreases have made Monte Carlo simulation readily accessible, and hence the popularity of Markov has declined. However it is still useful for the analysis of multiple state systems and those that exhibit strong dependency between components and is used in commercial AR&M modelling tools that use state transition diagrams to calculate reliability and maintainability values for complex systems.

**1.1.4** Other reasons for the lack of popularity of Markov are:

- The fact that it is not an easy tool for engineers to apply and to explain to others.
- Monte Carlo simulation programs exist that not only model complex systems but also the use of the systems in complex operational scenarios.

| MARKOV ANALYSIS | | | MONTE CARLO SIMULATION |
|---|---|---|---|
| ADVANTAGES | DISADVANTAGES | SOURCE | |
| A1. <u>Simplistic Modelling Approach</u>. The models are simple to generate although they do require a more complicated mathematical approach. | D1. Vast increase in number of states as the size of the system increases. The resulting diagrams for large systems are generally very extensive and complex, difficult to construct and computationally extensive. | A1: Reference 1<br>D2: Reference 1 | Very flexible. There is virtually no limit to the analysis. Can generally be easily extended and developed as required. |
| A2. <u>Redundancy Management Techniques</u>. System reconfiguration required by failures is easily incorporated in the model. | D2. Markov modelling of redundant repairable systems with automatic fault detection and one repair crew is flawed. This is because although random failure is a Markov process, repair of multiple failures is not a Markov Process. The mathematical discrepancy may be overcome by using a dedicated repair crew per equipment, but this does not normally correspond to real life support strategies. | A2: Reference 1<br>D2: Reference 2 | Redundancy management easily handled. |
| A3. <u>Coverage</u>. Covered (detected and isolated) and uncovered (undetected) failures of components are mutually exclusive events, not easily modelled using classical techniques but readily handled by Markov mathematics. | | A3: Reference 1 | Covered and uncovered failures of components readily handled. |
| A4. Complex maintenance options can readily be modelled. | D4. Can only deal with constant failure rates and constant repair rates - the latter being unrealistic in real, operational systems for many reasons including, for example, changing physical conditions and variations in maintenance skills. However, if the MTTR is very much shorter than the MTTF, then this shortcoming rarely introduces serious inaccuracy in the final computed system parameters. | A4: Reference 3<br>D4: Reference 3 | Complex maintenance options can readily be modelled.<br>A wide range of distributions including empirical distributions can be handled. |
| | D5. Future states of the system are independent of all past states except the immediately preceding one, which implies that a repair returns the system to an "as new" condition. | 5D: Reference 4 | Can incorporate distributions that embrace "wear-out" conditions. |
| A6. <u>Complex Systems</u>. Many simplifying techniques exist which allow the modelling of complex systems. | | A6: Reference 1 | Complex systems readily handled. |
| A7. <u>Sequenced Events</u>. Markov modelling easily handles the computation of the probability of an event resulting from a sequence of sub-events. This type of problem does not lend itself well to classical techniques. | | A7: Reference 1<br>Reference 3 | Sequenced events readily handled. |

Table 1: Advantages and limitations of Markov Analysis compared to Monte Carlo Simulation

# 2 MARKOV ANALYSIS

## 2.1 General

**2.1.1** Markov analysis is a complex subject with many applications outside the field of R&M engineering. Most technical libraries will have several books on the subject. It is covered in this manual since it is an analysis method that can be applied to certain reliability problems. The method is based on an analysis of the transitions between system states. Markov analysis is illustrated by example in Section 3 of this Chapter.

**2.1.2** The basis of a Markov model is the assumption that the future is independent of the past, given the present. This arises from the study of Markov chains – sequences of random variables in which the future variable is determined by the present variable but is independent of the way in which the present state arose from its predecessors. Markov analysis looks at a sequence of events and analyses the tendency of one event to be followed by another. Using this analysis, it is possible to generate a new sequence of random but related events, which appear similar to the original.

**2.1.3** A Markov chain may be described as homogeneous or non-homogeneous. A homogeneous chain is characterised by constant transition times between states. A non-homogeneous chain is characterised by transition rates between the states that are functions of a global clock, for example, elapsed mission time. In R&M analysis a Markov model may be used where events, such as the failure or repair of an item can occur at any point in time. The model evaluates the probability of moving from a known state to the next logical state, i.e. from everything working to the first item failed, from the first item failed to the second item failed and so on until the system has reached the final or totally failed state.

## 2.2 System States and Truth Tables

**2.2.1** A SYSTEM STATE is a particular combination of the states of the elements comprising the system. For example, for a system comprising two elements ‘x’ and ‘y’, each element capable of taking one of two states (up or down), there are 4 possible system states:

(a) x up y up

(b) x up y down

(c) x down y up

(d) x down y down

**2.2.2** In general, if elements can be in one of ‘m’ states, the number of possible system states for an ‘n’ element system is $m^n$.

**2.2.3** The list of all possible system states in terms of the element states is called the TRUTH TABLE for the system; (a) to (d) above comprise a truth table. Each line of the truth table can be identified with a system condition, up or down (or degraded). For example, if elements ‘x’ and ‘y’ were in series, then (a) would be a system up state, and (b), (c) and (d) would be down states. If ‘x’ and ‘y’ were in a redundant configuration, then states (a), (b) and (c) would represent system up states and (d) the system down state.

## 2.3 Markov Analysis and Transition State Diagrams

**2.3.1** Markov analysis computes the rates at which transitions occur between system states from such parameters as the element failure rates and/or repair rates. This is then used to compute system parameters such as MTBF, reliability, availability, etc. The mathematics is illustrated in Section 3 of this Chapter.

**2.3.2** Generally Markov analysis in reliability applications is confined to the situation where the distribution of element failure and repair times is negative exponential. It is also generally assumed that two elements cannot change their states simultaneously. Thus, for example, the 2 element system of paragraph 2.2.1 cannot change from state (a) to state (d) at one time, since this would require ‘x’ and ‘y’ to fail simultaneously. Possible transitions between system states and the rate at which they occur are indicated in Transition State Diagrams, as shown in Section 3.

**2.3.3** Despite these limitations the technique is of value since such assumptions are frequently made in reliability work, and it can handle situations where the failure and repair time distributions of the element are not independent (as is the case with standby systems).

# 3 EXAMPLE OF MARKOV METHODS FOR ANALYSING THE RELIABILITY OF COMPLEX SYSTEMS

## 3.1 Introduction

**3.1.1** This Section describes, by means of an example, a method of analysing the reliability of complex systems. Although the example chosen is of a non-repairable standby system, it is relatively straightforward to adapt the method to model repairable systems.

**3.1.2** The analysis technique employs some of the ideas used in the analysis of Markov Processes. An important assumption required by (and limitation of) the method is that the failure rates of the elements comprising the system are constant (for repairable systems, the repair rates must also be constant), i.e. the failure time (and repair time) distributions must be negative exponential. In real time operational situations this may be unrealistic and care must be taken when applying this technique.

## 3.2 The Analysis Method

**3.2.1** Consider the system shown in, comprising 2 elements in standby redundancy. Let the failure rate of element 1 be $\lambda_1$, and the failure rates of element 2 be $\lambda_2$ in the operational state and $\lambda_{p2}$ in the standby state.

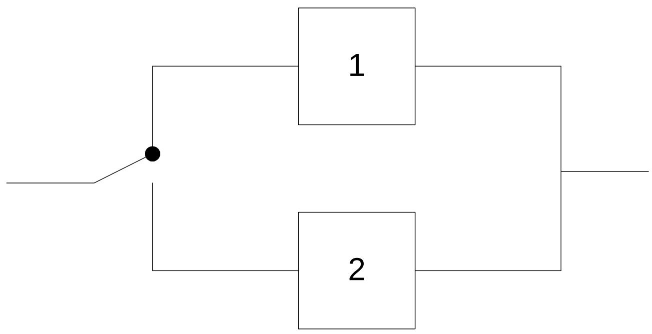


Figure 1: A Two Element Standby Redundancy System

**3.2.2** Now this system can occupy one of four states:

(1) 1 and 2 up ≡ $(1, 2)$

(2) 1 down, 2 up ≡ $(\bar{1}, 2)$

(3) 1 up, 2 down ≡ $(1, \bar{2})$

(4) 1 down, 2 down ≡ $(\bar{1}, \bar{2})$

The system states may be represented diagrammatically as:

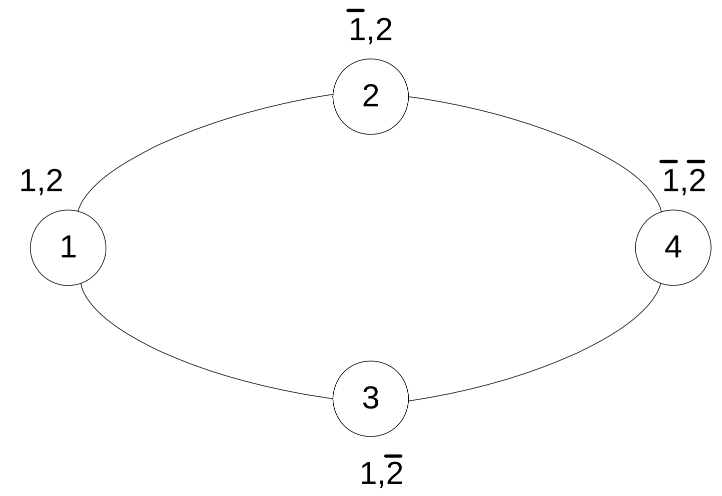


Table 2: The System States

**3.2.3** Let the probability of the system being in state 1 at time $t = p_1(t)$

Consider the probability of the system being in state 1 at time $t + \Delta t$.

The following relationships hold (assuming that $\Delta t$ is sufficiently small that the probability of two or more transitions taking place in the time interval $\Delta t$ is negligible).

$$p_1(t + \Delta t) = \lfloor 1 - (\lambda_1 + \lambda_{p2})\Delta t \rfloor p_1(t)$$

(since the probability of being in state (1) at $t + \Delta t$ is the product of the probability that the system was in state (1) at time $t$ and the probability that neither of elements 1 and 2 failed in $\Delta t$.

Similarly:

$$p_2(t + \Delta t) = \lambda_1 \Delta t p_1(t) + (1 - \lambda_2 \Delta t) p_2(t)$$

$$p_3(t + \Delta t) = \lambda_{p2} \Delta t p_1(t) + (1 - \lambda_1 \Delta t) p_3(t)$$

---------------(1)

$$p_4(t + \Delta t) = \lambda_2 \Delta t p_2(t) + \lambda_1 t p_3(t) + p_4(t)$$

These diagrams can be illustrated by a diagram called the TRANSITION STATE DIAGRAM for the system.

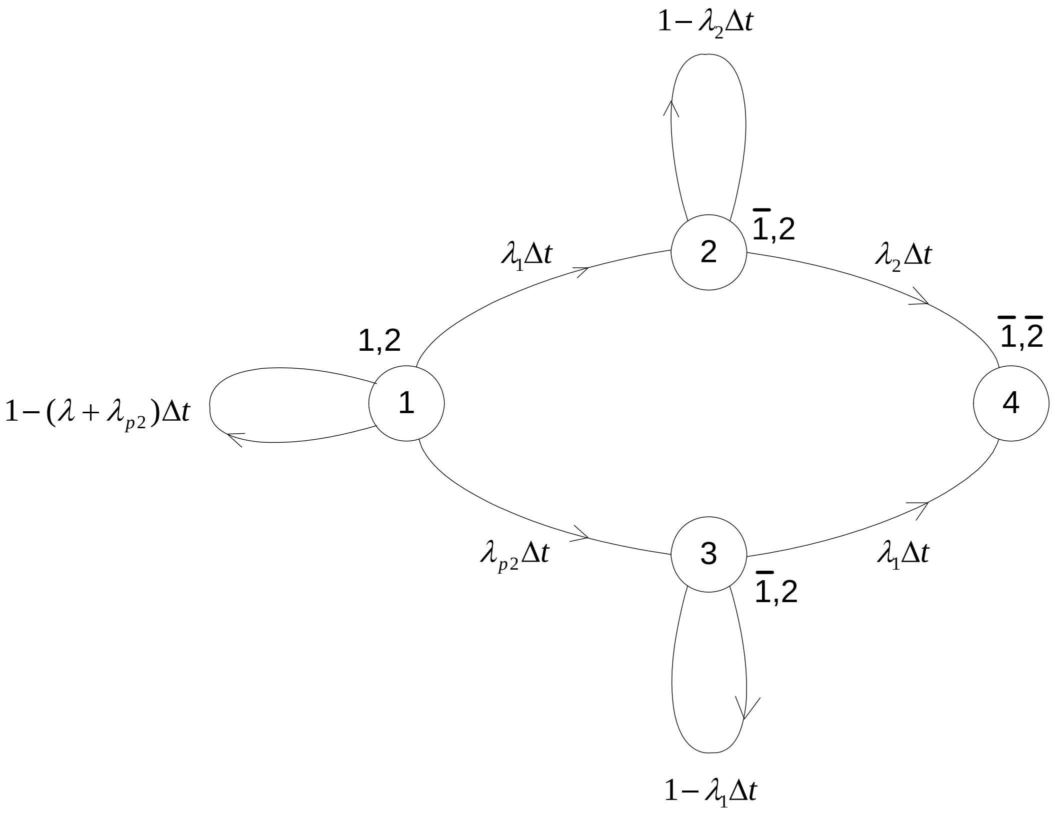


Figure 3: Transition State Diagram

Notes: (i) the arrows indicate the directions of transitions between system states.

(ii) 4 is an ‘absorbing state’, i.e. once entered, the system cannot leave it.

**3.2.4** Equation (1) can be re-arranged as follows:

$$\left.\begin{aligned}
\frac{p_1(t+\Delta t)-p_1(t)}{\Delta t} &= -(\lambda_1+\lambda_{p2})\,p_1(t) \\[2ex]
\frac{p_2(t+\Delta t)-p_2(t)}{\Delta t} &= \lambda_1 p_1(t)-\lambda_2 p_2(t) \\[2ex]
\frac{p_3(t+\Delta t)-p_3(t)}{\Delta t} &= \lambda_{p2} p_1(t)-\lambda_1 p_3(t)
\end{aligned}\right\} \quad \text{---------------(2)}$$

$$\frac{p_4(t+\Delta t)-p_4(t)}{\Delta t} = \lambda_2 p_2(t)+\lambda_1 p_3(t)$$

Taking the limit as $\Delta t \to 0$ yields:

$$\left.\begin{aligned}
\dot{p}_1(t) &= -(\lambda_1+\lambda_{p2})\,p_1(t) \\
\dot{p}_2(t) &= \lambda_1 p_1(t)-\lambda_2 p_2(t) \\
\dot{p}_3(t) &= \lambda_{p2} p_1(t)-\lambda_1 p_3(t) \\
\dot{p}_4(t) &= \lambda_2 p_2(t)+\lambda_1 p_3(t)
\end{aligned}\right\} \quad \text{---------------(3)}$$

Where $\dot{p}_1(t) = \dfrac{d(p_1(t))}{dt}$

**3.2.5** These differential equations can be solved by taking Laplace Transforms, as shown in Appendix 2. The solutions are:

$$\left.\begin{aligned}
p_1(t) &= e^{-(\lambda_1+\lambda_{p2})t} \\[1ex]
p_2(t) &= \left[\frac{\lambda_1}{\lambda_1+\lambda_{p2}-\lambda_2}\right]e^{-\lambda_2 t} - \left[\frac{\lambda_1}{\lambda_1+\lambda_{p2}-\lambda_2}\right]e^{-(\lambda_1+\lambda_{p2})t} \\[1ex]
p_3(t) &= e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_{p2})t} \\[1ex]
p_4(t) &= 1-e^{-\lambda_1 t} - \left[\frac{\lambda_1}{\lambda_1-\lambda_2+{}_{p2}}\right]e^{-\lambda_2 t} + \left[\frac{\lambda_1}{\lambda_1-\lambda_2+\lambda_{p2}}\right]e^{-(\lambda_1+\lambda_{p2})t}
\end{aligned}\right\} \quad \text{-----------(4)}$$

**3.2.6** The reliability of the system at time t is the probability that the system is in states (1), (2) or (3):

i.e.

$$R(t) = p_1(t) + p_2(t) + p_3(t)$$

$$= 1 - p_4(t)$$

$$= e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2 + {}_{p2}} e^{-\lambda_2 t} - \frac{\lambda_1}{\lambda_1 - \lambda_2 + \lambda_{p2}} e^{-(\lambda_1 + \lambda_{p2})t}$$

**3.2.7** There are other methods of analysing the system described above. One advantage of this method however is that given that the system can be described in a transition state diagram such as that shown in Fig 3, the method is completely general and can be incorporated in a computer program. Theoretically, a system of any complexity may be analysed in this way, although in practice there will be limitations imposed by the size of the system and the computer program to analyse it (i.e. for a system comprising n elements there will be $2^n$ system states – if, however, some of the elements are identical, this number may be reduced).

**3.2.8** As stated in the introduction, the method can be adapted easily to the analysis of repairable systems. Consider the same 2 element standby redundancy system where elements 1 and 2 have repair rates $\left(= \frac{1}{MTBF}\right)\mu_1$ and $\mu_2$ respectively. The transition state diagram is now:

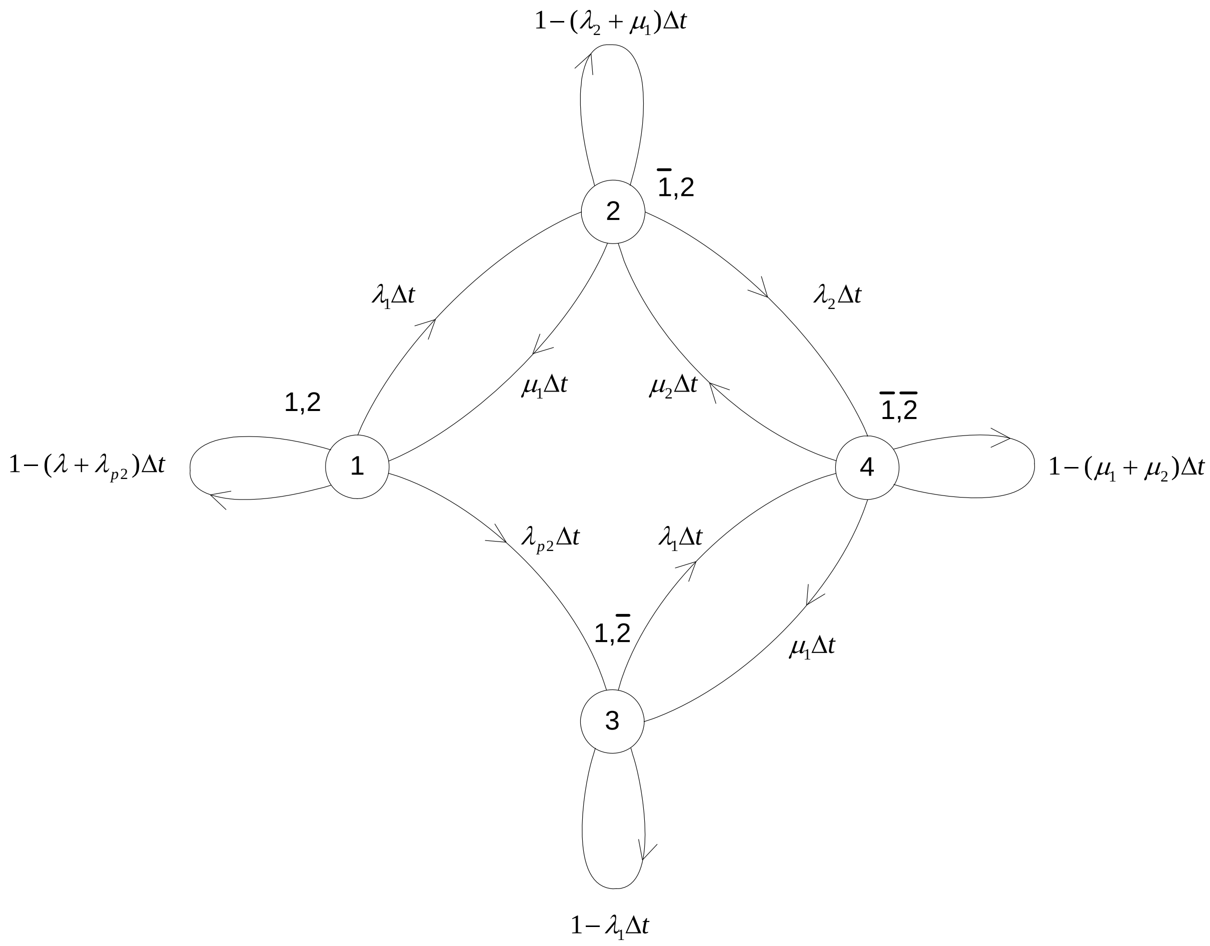


Figure 4: Transition State Diagram for a Repairable Standby System

Note: There is no transition allowed from 3 to 1. This arises from the assumption that a passive failure of element 2 (i.e. a transition from 1 to 3) will not be detected until the element is required for operation (i.e. when element 1 fails). Hence no repair action is taken on the passive failure of element 2, and hence there can be no transition from state 3 to 1.

**3.2.9** The transitional probability equations similar to (1) can now be set up and solved as before. It should be noted that, in this case, because we are dealing with a repairable system, the sum given by $p1(t) + p2(t) + p3(t)$ represents the availability and not the reliability of the system at time t. The reliability of the system (i.e. its probability of survival to time $t$ – $R(t)$) may be calculated by modifying the transition state diagram to that shown in Fig 5. The reason for treating the calculation of reliability (R(t)) in this way arises from the fact that reliability is the probability that a system will not fail in a given period of time.

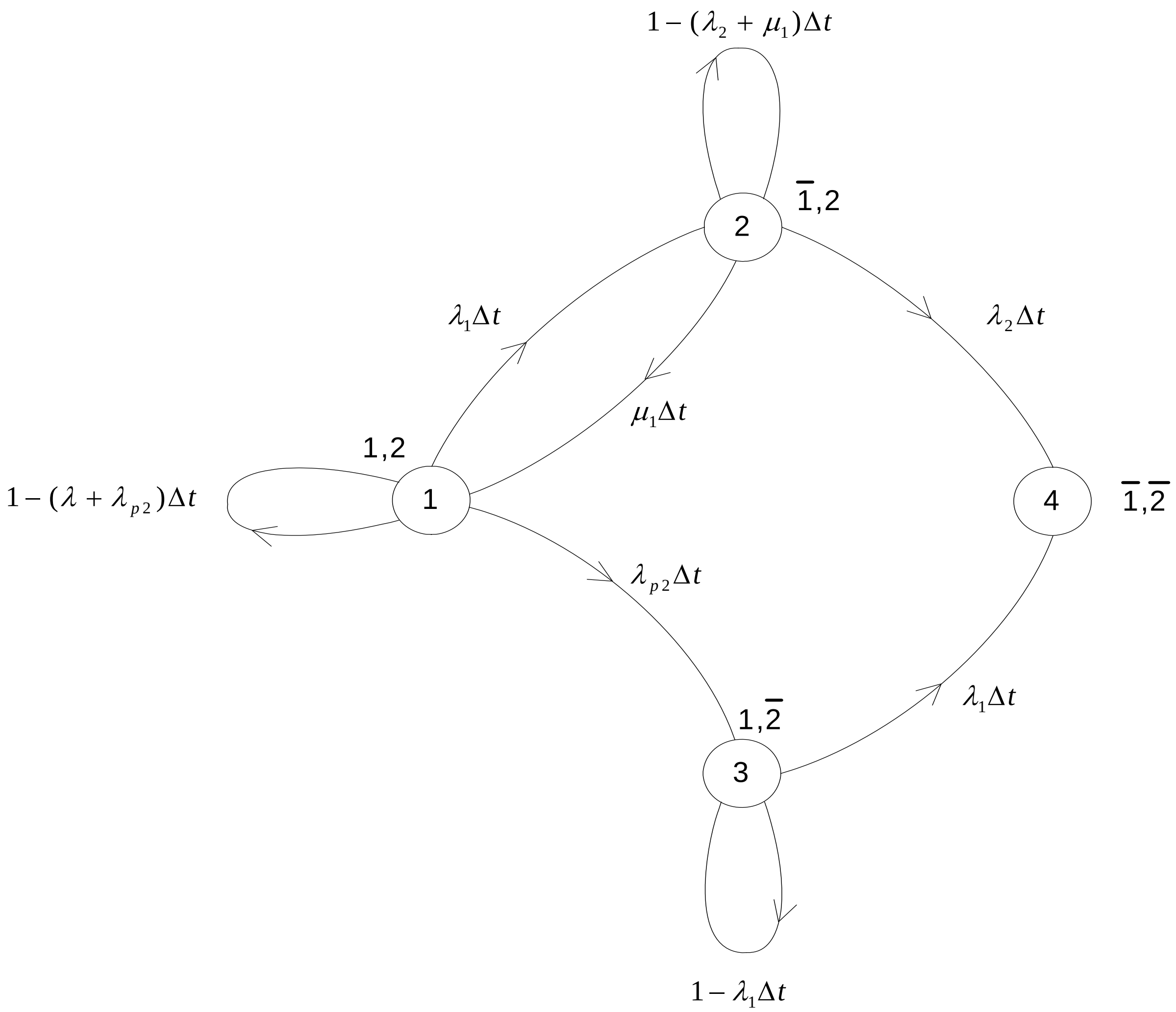


Figure 5: Transition State Diagram for a Repairable Standby System to Calculate its Reliability (R(t))

A general rule for the construction of transition state diagrams is that for availability calculations, all allowable repair transitions should be included, whereas for reliability calculations the system 'down' states, e.g. state 4 in Fig 5, should be treated as absorbing states, i.e. once entered, the system cannot leave them.

# 4 SOLUTION OF THE DIFFERENTIAL EQUATIONS DESCRIBING THE STATE PROBABILITIES, USING LAPLACE TRANSFORMS

## 4.1 Introduction

**4.1.1** This paragraph describes how to solve the differential equations labelled (3) in paragraph 3, (but (1) here). The equations were:

$$\left.\begin{aligned} \dot{p}_1(t) &= -(\lambda_1 + \lambda_{p2})\, p_1(t) \\ \dot{p}_2(t) &= \lambda_1 p_1(t) + \lambda_2 p_2(t) \\ \dot{p}_3(t) &= \lambda_{p2} p_1(t) + \lambda_1 p_3(t) \\ \dot{p}_4(t) &= \lambda_2 p_2(t) + \lambda_1 p_3(t) \end{aligned}\right\} \text{---------------(1)}$$

where $\dot{p}_1(t) = \dfrac{d(p_1(t))}{dt}$

**4.1.2** The method of solution adopted here is that using Laplace Transforms. The basis of the method is not described here, except to say that it converts differential equations to algebraic forms by means of a transformation discovered by Laplace. The algebraic equations may then be easily solved, the inverse of the transformation applied to obtain the final solution. Some Laplace Transforms are given in Table 1.

## 4.2 Solution

**4.2.1** Let $L_i(s)$ be the Laplace Transforms of $\dot{p}_i(t)$ * and take Laplace Transforms of equations (1).

Now the Laplace Transform of $\dot{p}_i(t)$ is given by $sL_i(s) - p_i(0)$. If, at t = 0, both elements 1 and 2 are up then the system will be in state 1 (paragraph 3.2.2),

i.e. $p_1(0) = 1, p_2(0) = p_3(0) = p_4(0) = 0$

* $L_i(s) = \int_0^\infty p_i(\tau) e^{-s\tau} d\tau$

$$\left.\begin{aligned} \therefore \quad sL_1(s)-1 &= (\lambda_1+\lambda_{p2})L_1(s) \\ sL_2(s) &= \lambda_1 L_1(s) - \lambda_2 L_2(s) \\ sL_3(s) &= \lambda_{p2} L_1(s) - \lambda_1 L_3(s) \\ sL_4(s) &= \lambda_2 L_2(s) + \lambda_1 L_3(s) \end{aligned}\right\} \text{--------------}\mathbf{(2)}$$

$$\left.\begin{aligned} \therefore \quad (s+\lambda_1+\lambda_{p2})L_1(s) &= 1 \\ (s+\lambda_2)L_2(s) &= \lambda_1 L_1(s) \\ (s+\lambda_1)L_3(s) &= \lambda_{p2} L_1(s) \\ sL_4(s) &= \lambda_2 L_2(s) + \lambda_1 L_3(s) \end{aligned}\right\} \text{--------------}\mathbf{(3)}$$

**4.2.2** For the object of the exercise is to solve these equations (3) algebraically for the $L_1(s)$, then take inverse Laplace Transforms to obtain solutions for the $p_i(t)$, (i = 1 to 4 in this case). In producing the expression of $L_1(s)$ it is necessary to put it into a form suitable for the inverse transformation. Such forms can be obtained from tables of Laplace Transforms (e.g. Table 1). In this case the suitable form is:

$$L_i(s) = \frac{A}{s+\alpha} + \frac{B}{s+\beta} + \ldots \text{ etc} \qquad \text{--------------(4)}$$

where *A*, *B*, $\alpha, \beta$ do not involve s.

**4.2.3** Thus from (3):

$$L_1(s) = \frac{1}{s+\lambda_1+\lambda_{p2}} \qquad \text{--------------(*)}$$

$$L_2(s) = \frac{\lambda_1}{(s+\lambda_2)(s+(\lambda_1+\lambda_{p2}))}$$

$L_2(s)$ is put into the form of equation (4) as follows:

$$\frac{1}{(s+\lambda_2)(s+\lambda_1+\lambda_{p2}))} = \frac{A}{s+\lambda_2} + \frac{B}{s+(\lambda_1+\lambda_{p2})} \qquad \text{--------------(5)}$$

Since this is an identity we have:

$$\lambda_1 = As + A(\lambda_1 + \lambda_{p2}) + Bs + B\lambda_2 \qquad \text{--------------(6)}$$

Equating coefficients of s on both sides yields:

$$0 = A + B \quad \text{i.e.} \quad A = -B$$

Inserting this in (6) yields:

$$\lambda_1 = A(\lambda_1 + \lambda_{p2}) - A\lambda_2$$

$$\therefore \qquad A = \frac{\lambda_1}{\lambda_1 + \lambda_{p2} - \lambda_2}$$

Therefore from (5)

$$L_2(s) = \frac{\lambda_1}{\lambda_1 + \lambda_{p2} - \lambda_2}\left[\frac{1}{s+\lambda_2} - \frac{1}{s + (\lambda_1 + \lambda_{p2})}\right] \qquad \text{--------------(*)}$$

Similarly

$$L_3(s) = \frac{\lambda_{p2}}{(s+\lambda_1)(s + (\lambda_1 + \lambda_{p2}))}$$

$$\therefore \qquad L_3(s) = \frac{1}{s+\lambda_1} - \frac{1}{s + (\lambda_1 + \lambda_{p2})} \qquad \text{--------------(*)}$$

The correct form for $L_4(s)$ is most easily obtained from the expression:

$$L_4(s) = \frac{1}{s} - L_1(s) - L_2(3) - L_3(s)$$

(which itself derives from the fact that $p_1(t) + p_2(t) + p_3(t) + p_4(t) = 1$)

$\therefore$

$$L_4(s) = \frac{1}{s} - \frac{1}{s+\lambda_1+\lambda_{p2}} - \frac{\lambda_1}{\lambda_1 + \lambda_{p2} - \lambda_2}\left[\frac{1}{(s+\lambda_2)} - \frac{1}{s + (\lambda_1 + \lambda_{p2})}\right] - \left[\frac{1}{s+\lambda_1} - \frac{1}{s + (\lambda_1 + \lambda_{p2})}\right]$$

$$\therefore L_4(s) = \frac{1}{s} - \frac{1}{s+\lambda_1} - \frac{\lambda_1}{(\lambda_1 + \lambda_{p2} - \lambda_2)}\left[\frac{1}{s+\lambda_2}\right] + \frac{\lambda_1}{(\lambda_1 + \lambda_{p2} - \lambda_2)}\left[\frac{1}{s+\lambda_1+\lambda_{p2}}\right] \qquad \text{---------(*)}$$

**4.2.4** The solution now comes from the equations labelled (*). The inverse Laplace Transform of $L_1(s)$ is $p_1(t)$ by definition, that of $A/(s+\alpha)$ is $Ae^{-\alpha t}$, and that of 1/s is 1 (see Table 1). Hence:

$$\left.\begin{aligned}
p_1(t) &= e^{-(\lambda_1+\lambda_{p2})t} \\
p_2(t) &= \left[\frac{\lambda_1}{\lambda_1+\lambda_{p2}-\lambda_2}\right]e^{-\lambda_2 t} - \left[\frac{\lambda_1}{\lambda_1+\lambda_{p2}-\lambda_2}\right]e^{-(\lambda_1+\lambda_{p2})t} \\
p_3(t) &= e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_{p2})t} \\
p_4(t) &= 1-e^{-\lambda_1 t} - \left[\frac{\lambda_1}{\lambda_1-\lambda_2+_{p2}}\right]e^{-\lambda_2 t} + \left[\frac{\lambda_1}{\lambda_1-\lambda_2+\lambda_{p2}}\right]e^{-(\lambda_1+\lambda_{p2})t}
\end{aligned}\right\} \text{--------------(7)}$$

**4.2.5** Thus the expressions labelled (7) above are the solutions of the differential equations labelled (1) in paragraph 4.1.1.

| x(t) | Laplace Transform of x(t) |
|---|---|
| 1 | $\frac{1}{s}$ |
| $\frac{t^{n-1}}{(n-1)!}$ (n a positive integer) | $\frac{1}{s^n}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $\frac{1}{a}\sin(at)$ | $\frac{1}{s^2+a^2}$ |
| $\cos(at)$ | $\frac{s}{s^2+a^2}$ |
| $\frac{1}{a}\sinh(at)(s >\|a\|)$ | $\frac{1}{s^2-a^2}$ |
| $\cosh(at)(s >\|a\|)$ | $\frac{s}{s^2-a^2}$ |
| $\frac{1}{2a^3}[\sin(at) - at.\cos(at)]$ | $\frac{1}{(s^2+a^2)^2}$ |
| $\frac{t}{2a}\sin(at)$ | $\frac{s}{(s^2+a^2)^2}$ |
| $\frac{d}{dt}(x(t))$ | s.L(s)=x(0) when L(s) is Laplace Transform of x(t) |
| $\frac{d^n}{dt^n}(x(t))$ | $s^n.L(s) - s^{n-1}x(0) - s^{n-2}\left[\frac{dx}{dt}\right]_0 ... - \left[\frac{d^{n-1}x}{dt^{n-1}}\right]_0$ where $\left[\frac{d^i x}{dt^1}\right]_0$ is the value of the $i^{th}$ derivative of x(t) at t=0 |

**Table 2: Table of Common Laplace Transforms**

Notes: (1) The Laplace Transform of x(t) is defined by: $L(s) = \int_0^\infty e^{-st} x(t)dt$

(2) Specialist books will provide more comprehensive tables of transforms.

# 5 RELATED DOCUMENTS

1. “Applicability of Markov Analysis Methods to Reliability, Maintainability and Safety.” by Norman B. Fuqua. Published in the Reliability Analysis Centre - Selected Topics in Assurance Related Technologies (START) Volume 10 No 2.
2. “Repairable Redundant Systems and the Markov Fallacy” by W G Gulland and “Reliability Assessments of Repairable Systems – Is Markov Modelling Correct?” by KGL Simpson and M Kelly. Published in Safety and Reliability Society Journal Volume 22 No 2.
3. British Standard 5760. *Reliability of systems, equipment and components.* Part 2:1994. Section 11
4. “Calculating probability of Failure of Electronic and Electrical Systems (Markov vs. FTA) by Vito Faraci Jr. Published in the Journal of the Reliability Analysis Center, Third Quarter 2001.