CHAPTER 9

AVAILABILITY DEMONSTRATION PLANS

CONTENTS

1 INTRODUCTION

1.1 The purpose of an Availability Demonstration is to establish that Availability performance of the equipment is better (or not) than a specified level, with some degree of confidence. It is assumed in this Chapter that the demonstration is carried out on a sample of a population. The sample is the equipment under test, which is representative of the total population of equipment produced, or intended for production.

1.2 The test equipment is considered as being either in an up or down state. The test is started with the equipment in the up state. After a period a failure occurs, causing the item to enter the down state, and a new cycle begins. PtCCh24 gives guidance to what down time should be considered relevant, depending on the Availability measure (i.e. Operational, Intrinsic or Effective Availability) which is specified in the requirement.

2 CONCEPTS AND TERMINOLOGY

2.1 General

2.1.1 Availability demonstrations are hypothesis tests (see PtDCh7). The essential first step is the definition of the hypothesis about the parameter to be tested.

2.1.2 There are three standard forms of test which can be used:

a) Fixed number of failures terminated

b) Fixed time terminated

c) Sequential.

2.2 Consumer's Risk

2.2.1 In any test of a sample there is a risk of accepting 'bad' equipment. Whatever the minimum standard the consumer sets, there is a finite probability that the sample will be such that a population worse than this standard will be accepted. This risk is quantified by the following definitions:

a) **Minimum Acceptable Availability** (A_β) is a value selected such that the risk associated of accepting the equipment of the value is tolerable.

b) **Consumer's Risk** (β) is the probability of accepting the equipment with a true Availability equal to the Minimum Acceptable Availability (A_β). Typically, β has a value of between 5% and 20%.

2.3 Producer's Risk

2.3.1 The reverse of Section 2.2.1 is also possible, in that there is also a risk of rejecting 'good' equipment. To ensure that the producer has a chance of passing the demonstration, he must design for a target Availability which is significantly greater than the Minimum

Acceptable Availability. The producer's target Availability (A_α) is dependent on the Consumer's and Producer's Risk, and the duration of the test.

2.3.2 The Producer's Risk (α) is the probability of rejecting the equipment with a true Availability equal to the target Availability. In other words, the probability of rejecting equipment with a true Availability greater than A_α will be less than α. Typically, α has a value between 5% and 20%.

2.4 Discrimination Ratio

2.4.1 The Discrimination Ratio (D) is a parameter which measures the 'power' of the test. The smaller the D, for a given α and β the steeper is the Operating Characteristic (OC) curve (see PtDCh7) and the better is the test. For an Availability Demonstration, the discrimination ratio is the ratio of the consumer's minimum acceptable unavailability $(1-A_\alpha)$ the producer's target unavailability $(1-A_\beta)$

$$D = \frac{1-A_\beta}{1-A_\alpha}$$

2.4.2 The selection of the producer's and consumer's risks will be a compromise between test accuracy and cost. The nearer the discrimination ratio is to one, the more expensive the test will be, but the better the test is at discriminating between good and bad.

2.4.3 Typical values for the Discrimination Ratio range between 1.5 and 3 with 2 being the most common. This is the same as other statistical demonstrations.

2.5 Distribution Assumptions

2.5.1 The test plans discussed in this Chapter are based on the following assumptions regarding up times and down times:

a) The up times are negative exponentially distributed

b) The down times are gamma distributed

The probability density functions of the negative exponential and gamma distributions are discussed in Pt D Ch 7. The test plans are only valid if these assumptions are true for the equipment under test. Any assumptions regarding the distribution of up times and down times should be based on previous experience, and in some cases supported by information about the physics of failure. If an equipment has a constant failure rate, the up times will be negative exponentially distributed. This is normally true for equipment which is not subject to early life failures or wear out failures, or repairable equipment made up of many components. Within a complex repairable equipment each component has its own failure characteristic, and may wear out and be replaced many times before others reach the wear out stage. Thus a complex repairable equipment never wears out, if components are replaced on failure. However, if no information is available about the equipment failure time distributions, the exponential may be used as an approximation, and checked using the Chi-Square or Kolmorgorov-Smirnov Test (see PtDCh7).

2.5.2 The Availability Demonstration test plans illustrated in this Chapter are based on the assumption of a gamma distribution for down times. BS 5760[1] states that this distribution is used for mathematical convenience and is not critical to the test plans, and that repair rate functions may be approximated to a gamma distribution, even if the true distribution is nearer to a log-normal. The shape of the gamma distribution is dependent on the shape parameter 'p'. When p = 1, the gamma distribution results in a negative exponential distribution and as p increases the probability density function tends to a lognormal distribution. Details of the gamma distribution and the effect of the shape parameter on the probability density function are illustrated in PtDCh7.

2.5.3 At the end of the demonstration, the up times must be scrutinised for trends. If the up times have an increasing or decreasing trend, the steady-state availability assumption will not be valid. This does not necessarily mean that the equipment is unacceptable. For example, if the equipment passes the test and the up times are tending to increase, obviously the equipment is acceptable. The consumer and producer may agree to undertake a trend and distribution test, or agree the assumptions before the demonstration test and thereby avoiding any disagreements of interpretation.

3 STATISTICAL TEST PLANNING

3.1.1 The statistical test plans result in either an accept or reject decision, as to whether the equipment satisfies the Availability requirement, given the associated risk levels, α and β. A test plan should be designed using the following steps:

a) The plan should include the Availability requirement, Availability measure (i.e. Operational, Intrinsic, Effective), the classification of states and times, failure definitions, the consequences of rejection and the acceptable value of steady state unavailability.

b) The consumer and producer need to agree the shape parameter 'p', for the assumed gamma distributed down times. Estimations of the mean up time and mean down time must be determined, such that a predicted equipment unavailability can be calculated.

c) Acceptable risk levels, α and β, are selected and the test parameters determined in accordance with the appropriate test plan.

4 DEMONSTRATION TEST PLANS

4.1 Fixed Number of Failures

4.1.1 The plan designs a demonstration for a minimum number failures (n) which satisfies the following inequalities:

$$F_{1-\alpha}(2pn,2n) \times F_{1-\beta}(2n,2pn) \leq D(1-U_0)/(1-DU_0) \qquad (1)$$

Where

$F_{1-\alpha}$ = the 1-α fractile of the F-distribution (see Table 1)

$F_{1-\beta}$ □ the 1-□ fractile of the F-distribution (see Table 1)

p = shape parameter for gamma distribution down times

n = number of failures

D = discrimination ratio

U_0 = minimum acceptable unavailability

4.1.2 The accept/reject decision is as follows:

Reject if $DT/UT > F_{1-\alpha}(2pn,2n) \times U_0 / (1 - U_0)$

Where :

DT = accumulated down time

UT = accumulated up time

Accept otherwise

4.1.3 *Example: The minimum acceptable Availability for an equipment is 90.0% and the producer's target Availability is 95.0%. Therefore, U_0 = 0.05 (producer's target unavailability) and U_1 = 0.1 (minimum acceptable unavailability). The down time (gamma distribution) shape parameter, p = 2. The consumer's risk (□ = 0.10 and the producer's risk (□) = 0.10.*

$$D(1 - U_0) / (1 - DU_0) = (2)(0.95)/(0.9) = 2.111$$

Determine the number of failures (n) which ensures the inequality equation [1] is satisfied. The result of $F_{1-\alpha}(2pn,2n) \times F_{1-\beta}(2n,2pn)$ can be determined from the F-distribution in Table 1. For example, for 5 failures $F_{1-\alpha}(2(2)(5),2(5)) \times F_{1-\beta}(2(5),2(2)(5)) = F_{1-\alpha}(20,10) \times F_{1-\beta}(10,20) = 2.2 \times 1.94 = 4.268$. Calculating relevant values:

Number of Failures (n)	*10*	*18*	*19*	*20*
$F_{1-\alpha}(2pn,2n) \times F_{1-\beta}(2n,2pn)$	*3.749*	*2.146*	*2.103*	*2.059*

Therefore, the test should run until 19 failures and restorations have occurred and reject if :

$$DT/UT > F_{1-\alpha}(2pn,2n) \times U_0 / (1 - U_0)$$

ie $DT/UT > 0.0784$.

0.80 fractiles

v_2 \ v_1	2	4	6	8	10	20	30	40	60	120
2	4.00	4.24	4.32	4.36	4.38	4.43	4.45	4.46	4.46	4.47
4	2.47	2.48	2.47	2.47	2.46	2.44	2.44	2.44	2.43	2.43
6	2.13	2.09	2.06	2.04	2.03	2.00	1.98	1.98	1.97	1.96
8	1.98	1.92	1.88	1.86	1.84	1.80	1.78	1.77	1.76	1.75
10	1.90	1.83	1.78	1.75	1.73	1.68	1.66	1.65	1.64	1.63
12	1.85	1.77	1.72	1.69	1.66	1.61	1.59	1.58	1.56	1.55
14	1.81	1.73	1.67	1.64	1.62	1.56	1.53	1.52	1.51	1.49
16	1.78	1.70	1.64	1.61	1.58	1.52	1.49	1.48	1.47	1.45
18	1.76	1.67	1.62	1.58	1.55	1.49	1.46	1.45	1.43	1.42
20	1.75	1.65	1.60	1.56	1.53	1.47	1.44	1.42	1.41	1.39
30	1.70	1.60	1.54	1.50	1.47	1.39	1.36	1.35	1.33	1.31
40	1.68	1.57	1.51	1.47	1.44	1.36	1.33	1.31	1.29	1.26
60	1.65	1.55	1.48	1.44	1.41	1.32	1.29	1.27	1.24	1.22
120	1.63	1.52	1.45	1.41	1.37	1.29	1.25	1.23	1.20	1.17

0.90 fractiles

v_2 \ v_1	2	4	6	8	10	20	30	40	60	120
2	9.00	9.24	9.33	9.37	9.39	9.44	9.46	9.47	9.47	9.48
4	4.32	4.11	4.01	3.95	3.92	3.84	3.82	3.80	3.79	3.78
6	3.46	3.18	3.05	2.98	2.94	2.84	2.80	2.78	2.76	2.74
8	3.11	2.81	2.67	2.59	2.54	2.42	2.38	2.36	2.34	2.32
10	2.92	2.61	2.46	2.38	2.32	2.20	2.16	2.13	2.11	2.08
12	2.81	2.48	2.33	2.24	2.19	2.06	2.01	1.99	1.96	1.93
14	2.73	2.39	2.24	2.15	2.10	1.96	1.91	1.89	1.86	1.83
16	2.67	2.33	2.18	2.09	2.03	1.89	1.84	1.81	1.78	1.75
18	2.62	2.29	2.13	2.04	1.98	1.84	1.78	1.75	1.72	1.69
20	2.59	2.25	2.09	2.00	1.94	1.79	1.74	1.71	1.68	1.64
30	2.49	2.14	1.98	1.88	1.82	1.67	1.61	1.57	1.54	1.50
40	2.44	2.09	1.93	1.83	1.76	1.61	1.54	1.51	1.47	1.42
60	2.39	2.04	1.87	1.77	1.71	1.54	1.48	1.44	1.40	1.35
120	2.35	1.99	1.82	1.72	1.65	1.48	1.41	1.37	1.32	1.26

0.95 fractiles

v_2 \ v_1	2	4	6	8	10	20	30	40	60	120
2	19.00	19.25	19.33	19.37	19.40	19.45	19.46	19.47	19.48	19.49
4	6.94	6.39	6.16	6.04	5.96	5.80	5.75	5.72	5.69	5.66
6	5.14	4.53	4.28	4.15	4.06	3.87	3.81	3.77	3.74	3.70
8	4.46	3.84	3.58	3.44	3.35	3.15	3.08	3.04	3.01	2.97
10	4.10	3.48	3.22	3.07	2.98	2.77	2.70	2.66	2.62	2.58
12	3.89	3.26	3.00	2.85	2.75	2.54	2.47	2.43	2.38	2.34
14	3.74	3.11	2.85	2.70	2.60	2.39	2.31	2.27	2.22	2.18
16	3.63	3.01	2.74	2.59	2.49	2.28	2.19	2.15	2.11	2.06
18	3.55	2.93	2.66	2.51	2.41	2.19	2.11	2.06	2.02	1.97
20	3.49	2.87	2.60	2.45	2.35	2.12	2.04	1.99	1.95	1.90
30	3.32	2.69	2.42	2.27	2.16	1.93	1.84	1.79	1.74	1.68
40	3.23	2.61	2.34	2.18	2.08	1.84	1.74	1.69	1.64	1.58
60	3.15	2.53	2.25	2.10	1.99	1.75	1.65	1.59	1.53	1.47
120	3.07	2.45	2.18	2.02	1.91	1.66	1.55	1.50	1.43	1.35

Table 1: F-Distribution

4.2 Fixed Time Test Plan

4.2.1 This test plan designs a demonstration with a duration (T) using the expression below. The test will result in duration of at least 15 times the estimated mean up time (m_u) which is required input for this plan. Pt C Ch 36 describes Reliability Prediction.

$$T = m_u\,(1+p^{-1})\left\{\frac{u_{1\text{-}\alpha}\sqrt{1-U_0} + \left[\dfrac{u_{1\text{-}\beta}\,(1-DU_0)\sqrt{D}}{\sqrt{1-U_0}}\right]}{(D-1)}\right\}^2 \qquad (2)$$

Where:

T	= test duration
m_u	= the estimated mean up time
p	= shape parameter for gamma distribution down times
$u_{1-\alpha}$	= the 1-α fractile of the standardised normal distribution
$u_{1-\beta}$	= the 1-β fractile of the standardised normal distribution
U_0	= minimum acceptable unavailability
D	= discrimination ratio

Values for $u_{1-\alpha}$ and $u_{1-\beta}$ are illustrated in Table 2.

$1-\alpha$ $1-\beta$	$u_{1-\alpha}$ $u_{1-\beta}$
0.80	0.842
0.90	1.282
0.95	1.645

Table 2: Standardised Normal Distribution

4.2.2 The accept/reject decision is as follows:

Reject if $DT/(UT + DT) > U_{lim}$

Where :

DT	= accumulated down time
UT	= accumulated up time

$$U_{lim} = U_0 \text{ x } \frac{\left[u_{1-\alpha} D(1-U_0) + u_{1-\beta}\sqrt{D}(1-DU_0)\right]}{\left[u_{1-\alpha}(1-U_0) + u_{1-\beta}\sqrt{D}(1-DU_0)\right]} \quad (3)$$

Accept otherwise.

4.2.3 *Example: Using the same values as for the example for the fixed number of failures plan in Section 4.2.3. Substituting these values into equation 2:*

$$T = m_u\,(1+2^{-1})\left\{\frac{1.282\sqrt{.95} + \left[\frac{1.282(1-2.0.05)\sqrt{2}}{\sqrt{.95}}\right]}{(1)}\right\}^2$$

$$= m_u\,(1.5)\left\{\frac{1.2495+1.6317}{1}\right\}^2$$

$$= 12.45\,m_u$$

Therefore, for an estimated mean up time of 100 hours, the test duration should be 1,245 hours. Reject if the DT/(UT+DT) is greater than:

$$U_{lim} = 0.05\left[\frac{1.282.2(.95) + 1.282\sqrt{2}(1-2.0.05)}{1.282(.95) + 1.282\sqrt{2}(1-2.0.05)}\right]$$

$$= 0.05\left[\frac{2.4358+1.6317}{1.2179+1.6317}\right]$$

$$= 0.07137$$

4.2.4 An alternative fixed time test plan, designs a demonstration with a test duration (T) based on the relationship between T and the discrimination ratio, D, given in Tables 3, 4 and 5. The test should only be applied when the ratio of mean down time and mean up time is less than 0.05. Pt C Ch 36 and Ch 37 detail Reliability and Maintainability prediction respectively.

4.2.5 The accept/reject decision is as follows:

Reject if DT/(DT+UT) > U_{lim}

where U_{lim} is a function of the duration (T) from Tables 4, 5 and 6.

Accept otherwise.

4.2.6 *Example: Using the same values as for the example for the fixed number of failures plan in Section 4.2.3. Using the values of D = 2, p = 2, $\alpha = \beta = 0.10$, Table 4 gives:*

T/m_u - 17.78.

U_{lim}/U_0 = 1.387

Assuming m_u = 100 hours, the test duration should be 1,778 hours. The producer's target unavailability (U_0) is 0.05, and therefore a reject decision is DT/(UT+DT) is greater than 0.06933.

T^*/m_u	p = 1					
	$\alpha = \beta = 0.05$		$\alpha = \beta = 0.10$		$\alpha = \beta = 0.20$	
	D	U_{lim}/U_0	D	U_{lim}/U_0	D	U_{lim}/U_0
1.0	24.73	3.92	15.01	2.91	7.37	1.86
1.2	20.01	3.64	12.39	2.75	6.30	1.83
1.4	16.88	3.42	10.64	2.63	5.56	1.80
1.6	14.66	3.24	9.38	2.52	5.03	1.77
1.8	13.01	3.10	8.44	2.44	4.63	1.74
2.0	11.73	2.98	7.70	2.36	4.31	1.71
2.5	9.52	2.75	6.42	2.22	3.74	1.65
3.0	8.12	2.58	5.59	2.11	3.37	1.61
3.5	7.15	2.45	5.01	2.03	3.10	1.57
4.0	6.44	2.34	4.58	1.96	2.90	1.54
5.0	5.46	2.19	3.98	1.86	2.61	1.49
6.0	4.82	2.07	3.58	1.78	2.42	1.45
7.0	3.46	1.99	3.29	1.72	2.28	1.42
8.0	4.02	1,92	3.07	1.67	2.17	1.40
9.0	3.75	1.86	2.90	1.63	2.08	1.37
10.0	3.53	1.81	2.76	1.60	2.01	1.36
15.0	2.88	1.65	2.33	1.49	1.78	1.29
20.0	2.53	1.56	2.10	1.42	1.65	1.26

Table 3: Discrimination Ratio (D) and Rejection Limit for Test Plan 3 with Shape Parameter (p) = 1

T^*/m_u	p = 2					
	$\alpha = \beta = 0.05$		$\alpha = \beta = 0.10$		$\alpha = \beta = 0.20$	
	D	U_{lim}/U_0	D	U_{lim}/U_0	D	U_{lim}/U_0
1.0	18.94	3.47	11.74	2.69	6.01	1.87
1.2	15.43	3.22	9.78	2.54	5.19	1.81
1.4	13.11	3.04	8.46	2.42	4.63	1.76
1.6	11.45	2.89	7.52	2.33	4.22	1.72
1.8	10.22	2.77	6.80	2.25	3.91	1.69
2.0	9.26	2.67	6.25	2.18	3.66	1.66
2.5	7.62	2.47	5.28	2.06	3.22	1.60
3.0	6.56	2.33	4.65	1.96	2.93	1.55
3.5	5.83	2.22	4.20	1.89	2.72	1.51
4.0	5.29	2.13	3.87	1.83	2.56	1.48
5.0	5.54	2.00	3.40	1.74	2.33	1.44
6.0	4.05	1.91	3.09	1.67	2.17	1.40
7.0	3.70	1.84	2.87	1.62	2.06	1.37
8.0	3.43	1.78	2.70	1.58	1.97	1.35
9.0	3.22	1.73	2.56	1.54	1.90	1.33
10.0	3.06	1.69	2.45	1.51	1.84	1.31
15.0	2.54	1.56	2.10	1.42	1.65	1.26
20.0	2.26	1.48	1.92	1.36	1.55	1.22

Table 4: Discrimination Ratio (D) and Rejection Limit for Test Plan 3 with Shape Parameter (p) = 2

T^*/m_u	p = 5					
	$\alpha = \beta = 0.05$		$\alpha = \beta = 0.10$		$\alpha = \beta = 0.20$	
	D	U_{lim}/U_0	D	U_{lim}/U_0	D	U_{lim}/U_0
1.0	15.37	3.14	9.64	2.52	5.05	1.83
1.2	12.72	2.93	8.19	2.38	4.48	1.77
1.4	10.87	2.77	7.15	2.27	4.04	1.72
1.6	9.54	2.64	6.38	2.19	3.71	1.68
1.8	8.55	2.54	5.80	2.11	3.45	1.61
2.0	7.78	2.45	5.36	2.05	3.25	1.61
2.5	6.47	2.28	4.57	1.94	2.89	1.55
3.0	5.62	2.16	4.06	1.85	2.65	1.51
3.5	5.03	2.07	3.70	1.79	2.47	1.47
4.0	4.59	1.99	3.43	1.73	2.34	1.44
5.0	3.98	1.88	3.04	1.65	2.15	1.40
6.0	3.58	1.80	2.79	1.60	2.02	1.36
7.0	3.29	1.73	2.60	1.55	1.92	1.34
8.0	3.07	1.68	2.46	1.51	1.84	1.32
9.0	2.90	1.64	2.34	1.48	1.78	1.30
10.0	2.76	1.61	2.25	1.46	1.73	1.28
15.0	2.32	1.49	1.96	1.37	1.57	1.23
20.0	2.09	1.42	1.80	1.32	1.48	1.20

Table 5: Discrimination Ratio (D) and Rejection Limit for Test Plan 3 with Shape Parameter (p) = 5

4.3 Sequential Test Plan

4.3.1 This test plan designs a demonstration with a duration which depends on the observed Availability of the equipment during test. After each repair, a decision is made as to whether the test should bed terminated or continued. The decision limits depend on the number of failures (n), that have occurred up to that instant of time.

The decision is based on the following rules:

Reject if DT/UT > Re(n) x $U_0/(1-U_0)$ (4)

Accept if DR/UT < Ac(n) x $U_0/(1-U_0)$ (5)

The test continues if neither of the above inequalities determine a reject or accept decision.

Where DT = Accumulated downtime

UT = Accumulated up time

U_0 = Producer's target unavailability

$$\mathrm{Re(n)} = \frac{[D - G(n)]}{p[G(n) - 1]} \text{ if } n > \ln \frac{[(1-\beta)/\alpha]}{p.\ln(D)}$$

else Re(n) = ∞

$$\mathrm{Ac(n)} = \frac{[D - H(n)]}{[H(n) - 1]} \text{ if } n > \ln \frac{[1-\beta/\alpha]}{p.\ln(D)}$$

else Ac(n) = 0.

Where $G(n) = D^{1/(1+p)} [\alpha/(1-\beta)]^{1/(n+np)}$

$H(n) = D^{1/(1+p)} [(1-\alpha)/\beta]^{1/(n+np)}$

4.3.2 *Example. Using the same data as before, i.e.* U_0 *= 0.05,* $\alpha = \beta$ *= 0.1, D = 2 and p = 2. Table 6 below illustrates an example of how a demonstration test can be monitored and an accept/reject decision determined. The first failure occurred after 60 hours, with 17 hours downtime, the second failure after 344 hours of uptime, followed by 28 hours downtime, etc. The table shows the values calculated for each parameter to make a decision, based on the rules in equations 4 and 5. Note, ND signifies no rejection is possible in this case, as the value for Re(n) is either negative or infinity.*

Table 6: Calculated Parameters for Sequential Test Plan Examples

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REFERENCES

1 BS 5760: Section 10.3: 1993. *Reliability of Systems, Equipment and Components. Part 10 - Guide to Reliability Testing. Section 10.3 - Compliance Test Procedures for Steady-State Availability.*

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