CHAPTER 4

MONTE-CARLO SIMULATION

CONTENTS

1 SIMULATION AND ANALYTIC MODELS

1.1 Analytic means exist for performing calculations using RBDs, but these calculations are only valid under certain restrictive assumptions such as independence of the blocks, no queuing for repair, etc. There are a number of modelling techniques which may be used to overcome these restrictions, and two such techniques are Markov analysis (see PtDCh38) and simulation, which is the subject of this Chapter.

1.2 Reliability models, by the nature of the process that they represent, have a significant probabilistic content. These processes usually involve the combination of two or more input random variables to produce output random variables. There are two approaches to the solution of these problems; one using analytical methods and the other using a technique called Monte-Carlo simulation.

1.3 In the analytical method, the probability distributions associated with the output random variables are *calculated* from the probability distributions associated with the input variables. In Monte-Carlo simulation, the value of a distributed parameter is selected by the generation of a random number, with the probability of a given value being determined by the association of random numbers to that variable. By repeating this process a large number of times, a picture of the distribution of the output random variable may be built up, from which estimates of the parameters of interest may be calculated, e.g. their mean, standard deviations, etc. A more detailed explanation of the Monte-Carlo simulation method, together with worked examples is given by Jones[2].

1.4 The two modelling methods are best explained by means of an example. Consider two items connected in series which have constant failure rates λ_1 and λ_2. The times to failure of individual items are governed by the negative exponential distribution with the probability density function (pdf) $f(x) = \lambda e^{-\lambda t}$, see PtDCh3. It can be shown by analytical methods (see PtDCh3) that the failure time distribution of the system comprising two items is also a negative exponential and the failure rate of the system is given by $\lambda_S = \lambda_1 + \lambda_2$. The MTBF of the system is given by $M_S = \frac{1}{\lambda_S} = \frac{1}{\lambda_1 + \lambda_2}$.

1.5 However, the system MTBF can also be estimated using Monte-Carlo simulation methods. A time is sampled from each failure time distribution for both elements, say T_1 and T_2. (Tocher[1] describes methods of how this can be performed in practice). The failure time of the system (T_S) is given in this case by $T_S = \min(T_1, T_2)$. By performing these operations a large number of times, the distribution of T_S is obtained and an estimate of the system MTTF is given by the mean of the sample values T_S.

2 COMPARISON OF METHODS

2.1 Each technique has it advantages and disadvantages; the main ones are listed in Table 1. Broadly, for complex systems that may be subject to change later, the Monte-Carlo method is preferred because of its flexibility. For simpler systems, or studies to get a ‘feel’ for a problem, analytical methods may suffice.

2.2 The decision as to whether the modeller should use analytical (e.g. deterministic equations) or simulation (i.e. Monte-Carlo) methods may be influenced by the following factors:

a) **Complexity.** Simulation often gives better physical visibility of a complex system analysis than a set of equations, aiding interpretation of the output.

b) **Scope.** For example, complex repair policies are easier to deal with in simulations than analytical models.

c) **Accuracy.** Although analytical models are deterministic, they usually involve simplifying assumptions to make the model analytically tractable. Such assumptions have to be justified.

d) **Future development.** If a model is likely to be further refined ad developed, an initial model that may be initially tractable analytically may not be so when further development requirements are placed. A simulation model may therefore be appropriate from the start.

e) **Application.** For quick look analysis, analytical models may be preferred, because of their speed of execution. The repeated running involved in Monte-Carlo simulation can cause long execution times before estimates of system parameters of interest are obtained.

<table>
<tr><th colspan="3">Simulation Method</th></tr>
<tr><th></th><th>Analytical</th><th>Monte-Carlo</th></tr>
<tr><td rowspan="3">Advantages</td><td>a. Gives exact results (given the assumptions of the model).</td><td>a. Very flexible. There is virtually no limit to the analysis. Empirical distributions can be handled.</td></tr>
<tr><td>b. Once the model is developed, output will generally be rapidly obtained.</td><td>b. Can generally be easily extended and developed as required.</td></tr>
<tr><td>c. It need not always be implemented on a computer – paper analyses may suffice.</td><td>c. Easily understood by non-mathematicians.</td></tr>
<tr><td rowspan="3">Disadvantages</td><td>a. Generally requires restrictive assumptions to make the problem tractable.</td><td>a. Usually requires a computer.</td></tr>
<tr><td>b. Because of a. it is less flexible than Monte-Carlo. In particular, the scope for extending or developing a model may be limited.</td><td>b. Calculations can take much longer than analytical models.</td></tr>
<tr><td>c. The model might only be understood by mathematicians. This may cause credibility problems if output conflicts with preconceived ideas of designers or management.</td><td>c. Solutions are not exact, but depend on the number of repeated runs used to produce the output statistics. That is, all outputs are estimates.</td></tr>
</table>

Table 1: Main Advantages and Disadvantages of Analytical and Monte-Carlo Simulation Models

3 STATISTICAL ACCURACY OF RESULTS

3.1 The Monte-Carlo simulation method is a type of sampling procedure, thus any output is not exact but a statistical estimate whose accuracy depends on the number of missions or failures generated. For example if mission parameters are of prime important (e.g. probability of mission survival failure free) then the number of missions to be simulated is the important parameter. The number of system failures generated is not necessarily important, e.g. if in 1000 mission simulated only 5 system failures are generated, mission reliability is none the less reasonably well established. However, if MTBF estimates are the prime consideration then a sufficient number of system failures must be simulated to yield the desired accuracy.

3.2 In the output of system R&M statistics derived from Monte-Carlo simulation models, confidence intervals are normally provided to indicate the accuracy of the parametric estimates.

3.3 The mathematics of selecting simulation parameters and deriving the confidence intervals is the same as that for reliability and Maintainability demonstrations, see PtCCh40 and PtCCh41.

4 HIGH RELIABILITY SYSTEMS

4.1 It is possible for simulation results to show widely different values for the same model when simulated with different random number seeds, or for the value of a parameter to be different compared with an estimate calculated analytically. Invariably, this situation arises with high reliability systems, simulated for relatively short mission times in which few system failures are generated.

4.2 For example, a system MTBF reported by an analysis program as 176 hours is a point estimate of the true population mean of times between failures. The accuracy of this figure, or its closeness to the true mean is indicated by the 95% confidence limits (CL) given by the program, and these should always be considered together with the mean. Hence a value of 176 with CL_L = 169 and CL_U = 183 (Confidence Interval = 14) is a better estimate of the true mean than 178 with CL_L = 149 and CL_U = 207 (Confidence Interval = 58).

4.3 As an example of what can happen with high reliability systems, let us look at a thingy modelled by a user who was particularly interested in system MTBF. She was looking at a 75-day mission so, quite reasonably, she set the mission time to 1800 hours, and ran for 5000 missions. The results were:

```
            System Reliability Results
            -------------------------
            Missions Simulated            5000
            Mission Time Simulated          1800.00
            Failure Free Missions         4988
            Total System Failures Recorded      12
                                         Lower        Upper
                   Estimate      95% C.L.      95% C.L.
MTBF                749999.       457261.     1.451518E+06
MTFF                749029.       456669.     1.449640E+06
AVAILABILITY    99.9999       99.9999       100.000
```

4.4 Only 12 system failures were generated, which resulted in a 95% CI around a mean of 750,000 of nearly 1 x 10^6. This was clearly a very bad estimate but it indicated a true MTBF that far exceeded the mission time and probably lay somewhere beyond 1 million hours. If the user was still grimly determined to get an MTBF figure for this ultra-reliable system she could have increased the mission time to, say, 1 x 10^6 hours, which would have given the following results:

System Reliability Results

Missions Simulated	1000
Mission Time Simulated	1000000.00
Failure Free Missions	518
Total System Failures Recorded	628

	Estimate	Lower 95% C.L.	Upper 95% C.L.
MTBF	1.592356E+06	1.497259E+06	1.687454E+06
MTFF	1.518071E+06	1.418889E+06	1.617253E+06
AVAILABILITY	100.000	100.000	100.000

4.5 A value for MTBF of 1.59 x 10^6 is now a better estimate because the confidence interval has been reduced by a factor of 10.

4.6 Clearly, when estimating MTBF for high reliability systems i.e. those with a high MTBF compared with typical operating times, it is necessary to generate enough system failures to achieve close confidence limits around the calculated mean. Therefore, real life operating times must be disregarded and the mission parameters chosen to ensure that enough system failures are generated.

LEAFLET 4/0

RELATED DOCUMENTS

1. K D Tocher. *The Art of Simulation.* The English Universities Press Ltd. 1973.

2. G T Jones. *Simulation and Business Decisions.* Penguin Books Ltd. 1972.

3. Patrick D T O'Connor. *Practical Reliability Engineering.* John Wiley & Sons.

Intentional blank page